Ch-1 Units and measurement.

* Introduction.

All quantities in terms of which laws of physics can be expressed and which can be measured directitly and indirectly are called Physical quantity.

Physical quantites classefied into two types

1) Fundamental Physical quantity
2) Derived Physical quantity.

→ Fundamental Physical quantity

The physical quantities which are independent of any other quantities are called Fundamental Physical quantity.

Eg:- Mass, length, time, temporature amount of substonce, luminous intensity (cd) and Electric current.

→ Derived Physical Quantity.

The physical quantity which can be derived from fundamental quantity are called Derived Quantity.

Eg:- force, accloration, speed, velocity, volume, etc.

→ units.

unit of measurement of physical quantity is the standard refrence of the same physical quantity which is used for comparition of the give physical quantity.

magnitude (numerical value)

1 Kg

unit

$$nu = \text{Constant}$$

n → numerical value.

u → units

$\boxed{n_1 u_1 = n_2 u_2}$

Lonvasion of one system to another system

→ System of units.

based on system of internotional there are four moin systems of units

i) CGS, MKS, FPS

→ CGS

C - Centimeter

G - Grom.

S - Second

→ MKS.

M - meter.

K - kilogram

S - second.

→ FPS

F - Foot

P - Pound

S - second.

→ Seven Fundamentol quontitys andunits

S.No	Physical Quantity	unit	Symbol
1	Mass	kilogram	kg
2	Length	Metre	m
3	time	Second	s
4	Temperature	Kelvin	K
5	Electric current	Ampere	A
6	luminous Intensity	Condela	cd
7	Amountof substance	Mole	mol

→ Derived units

Physical quantity	Derived unit	Symbol of unit.
Force.	Newton	$N = kg\ m/s^2$
Energy, Work, heat	Joule	$J = Nm = kg\ m^2/s^2$
Power	Watt	$W = J/s = N\ m/s = kg\ m^2/s^3$
Pressure, stress	Pascal.	$Pa = N/m^2 = kg/ms^2$
Frequency	hertz	$Hz = s^{-1}$
electric charge	Coulomb	$C = As$
electric potential	Volt	$V = J/C$
capacitance	Farad	$F = coulomb/volt = A^2 s^2/Nm$
Resistance.	ohm	$\Omega = V/A = J/A^2 s$
self inductance	Henry	$H = J/A^2 = Nm/A^2$
Magnetic flux	weber	Wb
Magnetic flux	Tesla	$T = Wb/m^2$

→ Multiples and submultiples of s
units.

Factor	Prefic	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

→ Plane angle is subtended at the center of the circle by an arc whos length is equal to the radias of the circle

si unit = radian $\quad d\theta = \frac{\text{arc length}}{\text{radius}}$

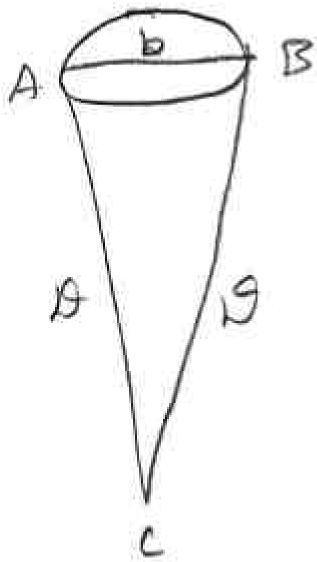
$$d\theta = \frac{ds}{r}$$

→ solid angle is sbtended at the center of a sphere by it surface, the area of which is = to the squre of the radias.

If $d\Omega$ be the solid angle subtended the center of the sphere. of radias (r) is $\boxed{d\Omega = \frac{ds}{r^2}} = \frac{4\pi \cancel{r^2}}{\cancel{r^2}}$ $\quad \boxed{d\Omega = 4\pi}$

→ measurment of large distances

The angle 'θ' b/w two directions along which the planet is vied at the two points is measured, which is called parallax angle.



$AC = BC$

$$\boxed{\theta = \frac{b}{\theta}}$$

$$\theta = \frac{b}{d}$$

b = diameter planet

θ = angle b/w 2 parallel sides in the planet

2.1 Ex:- From two diametrically opposite points A and B on earth, moon is observed. The angle θ subtended at the moon by the two directions of observation is $1^{\circ}54'$ (1.9°). If radius of earth is 0.638×10^{7} m, find the distance of the moon from the earth.

→ General Guidlines for using symbols for SI units/Prefixes

1) Unit name is never capitalized (I). units. symbols starts with capital letter only. 'y' unit name is to honour a person.
ex:- (5N), (1 D), (1c), (1F)
Newton, Diaptor, Columb, Faret.
(Except 1L)

2) Symbols for units do not contain any final fullstop at the end of recommended letter and remain unaltered in the plural. using only singular form of the unit.

→ Example :- for a length of 25 centimetres the unit symbol is written as 25 cm and not 25cm. or 25 cms or 25cms, etc.

3) Use of solidus(/) is recommended only for indicating a division of one letter unit symbol by another unit symbol. Not more than one solidus is used.
Ex:- m/s^2 or $m\ s^{-2}$ (with a spacing b/w m and s^{-2}) but m/s/s : J/kmol or $J\ K^{-1}\ mol^{-1}$, but not J/K/mol : etc

4) $1(M)W \rightarrow 10^6 W$
$1\ nm \rightarrow 10^{-9} m$
$1\ A^{\circ} \rightarrow 10^{-10} m$
$1\ PF \rightarrow 10^{-12} F$

5) The use of double prefixes is avoided when single prefixes are available.
Ex :- 10^{-9} m = 1 nm (nanometre), but not 1 mμm (millimicrometre).

6) The use of combination of unit and the symbole for units is avoided when the physical quantity is expressed by combining two or more units.

Ex:- $J\ mol^{-1}\ k^{-1}$ ✓

Joule mol^{-1} kelvin ✗

→ Dimensions

Dimensions of the unit of a physical quantity are the powers to which the units of the fundometal quantities mustbe raised to completly represent it.

Ex:- $F = [MLT^{-2}]$

$kg\,ms^{-2}$

The dimentional formula of a ~~phys~~ physical quntinty is ~~exprest~~ expressed in the form of [M],[L],[T],[K][I] or [A], Mass, Length, Time, temparature, electric current respectively.

→ Limitations on the dimensional Analysis.

i) Proportionality constants cannot be determined by dimensional analysis

ii) If a physical quantity depends upon more than three physical quantities, the relationship connot be derived easily

iii) Formulae containing non-algebraic functions cannot be derived. (sin, cos, log, exponential, etc.)

iv) Formuloe which contain two or more terms in the RHS cannot be derived

$$e.g = s = ut + \frac{1}{2} at^2$$

v) Dimensional analysis does not differatiate b/w a scalar and vector quantity

M → Mass
L → Length
T → time

K → Temp
A → Electric current

→ ~~Eq.~~ Dimensional formuale

1) Mass → kg → $[M^1 L^0 T^0]$

2) distance → m → $[M^0 L^1 T^0]$

3) second → S → $[M^0 L^0 T^1]$

4) velocity → m/s → $[M^0 L^1 T^{-1}]$
ms^{-1}

5) Accelaration → m/s^2 → $[M^0 L^1 T^{-2}]$
ms^{-2}

6) Area → m^2 → $[M^0 L^2 T^0]$

7) Volume → m3 → $[M^0 L^3 T^0]$

8) density → $\frac{m}{V} = \frac{kg}{m^3}$ → $[M^1 L^{-3} T^0]$

9) specific gravity → No dimensions.

10) Speed → $\frac{x}{t} = \frac{m}{s}$ $[M^0 L^1 T^{-1}]$

11) Acc^{-on} due to gravity → $9.8ms^{-2}$ → $[M^0 L^1 T^{-2}]$

12) momentum = $m \times V \rightarrow kgms^{-1} \rightarrow [M^1L^1T^{-1}]$

13) Force $\rightarrow kgms^{-2} \rightarrow [M^1L^1T^{-2}]$

14) Impulse $\rightarrow F \times t \rightarrow kgms^{-1}\ [M^1L^1T^{-1}]$

15) Work $\rightarrow F \times x \rightarrow kgm^2s^{-2} \rightarrow [M^1L^2T^{-2}]$

16) kinetic energy $\rightarrow \frac{1}{2}mv^2 \rightarrow kg\,(ms^{-1})^2$

$kgm^2s^{-2} \rightarrow [M^1L^2T^{-2}]$

17) Potential energy $\rightarrow mgh \rightarrow kgm^2s^{-2} \rightarrow [M^1L^2T^{-2}]$

18) moment of couple $\rightarrow F \times m \rightarrow kgm^2s^{-2} \rightarrow [M^1L^2T^{-2}]$

19) Torque $\rightarrow F \times 1^r$ dist $\rightarrow kgms^{-2} \times m \rightarrow [M^1L^2T^{-2}]$

20) Power $\rightarrow \frac{W}{t} = \frac{F \times x}{t} = kgm^2s^{-3} \rightarrow [M^1L^2T^{-3}]$

21) Pressure $\rightarrow \frac{F}{A}\ \frac{kgms^{-2}}{m^2} \rightarrow kgm^{-1}s^{-2} \rightarrow [M^1L^{-1}T^{-2}]$

22) Thrust = Pressure $\times A = kgm^{-1}s^{-2} \times m^2 \rightarrow [M^1L^1T^{-2}]$

23) Stress = $\frac{F}{A} = \frac{kgms^{-2}}{m^2} \rightarrow [ML^{-1}T^{-2}]$

24) Strain = No dimensional formula strain.

→ check weather the dimentional formula of each side correct or not

1) $V = u + at$

LHS = RHS

$V = u$

$ms^{-1} = ms^{-1}$

$[LT^{-1}] = [LT^{-1}]$

$$\left\{ \begin{aligned} at &\to ms^{-2} \times s^{+1} \\ &= ms^{-1} \\ &= LT^{-1} \end{aligned} \right\}$$

$V = at$

$[LT^{-1}] - [LT^{-1}]$

LHS = RHS

2) $F = \frac{mv^2}{r^2}$

$r \to$ radius

$v \to$ velocity

$m \to$ mass

$F = \frac{mv^2}{r}$

LHS = F

$= kg\,ms^{-2}$

$= [ML, T^{-2}]$

RHS $= \frac{mv^2}{r^2}$

$= \frac{kg\,(ms^{-1})^2}{m^2}$

$$= \frac{kg\,\cancel{m^2}\,s^{-2}}{\cancel{m^2}} = kg\,s^{-2}$$

$$RHS = [M^1 L^0 T^{-2}]$$

$$LHS \neq RHS.$$

3) $F = \frac{GMm}{r^2}$ ~~$GM\,m_2$~~

$M \& m \rightarrow$ masses

$r \rightarrow$ radius.

$F \rightarrow$ force

$G \rightarrow$ constant.

$LHS = RHS.$

$LHS = F$

$\Rightarrow kg\,m\,s^{-2}\ [M^1 L^1 T^{2}]$

$RHS \Rightarrow \frac{G \cdot Mm}{r^2}$

$\Rightarrow kg^2\,m^{-2}\ [M^2 L^{-2}]$

$$\boxed{LHS \neq RHS}$$

4) $v^2 = u^2 + 2as.$

$(ms^{-1})^2 = [ms^{-2}] \times [m]$

$[m^2 s^{-2}] = [m^2 s^{-2}]$

$[M^0 L^2 T^{-2}] = [M^0 L^2 T^{-2}]$

LHS = RHS

→ to convert unit of a physical quantity to one system to another system.

Newton → dyne.

$1N = 1\,kg\,ms^{-2}$

$= 1000g \times 100cm \times 1s^{-2}$

$= 10^3 \times 10^2\,g\,cms^{-2}$

$= 10^5\,g\,cms^{-2}$

$\boxed{1N = 10^5 \text{dyne}}$

$nu = constant$ $\quad u_1 \rightarrow [m_1^a L_1^b T_1^c]$

$n_1 u_1 = n_2 u_2$ $\quad u_2 \rightarrow [m_2^a L_2^b T_2^c]$

$n_2 u_2 = n_1 u_1$

$$n_2 = n_1 \frac{u_1}{u_2}$$

$$n_2 = n_1 \left[\frac{m_1}{m_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

→ Convert the unit of workdone from MKS to CGS system

Workdone → J → erg.

$1J = n\,erg.$

$$n = \frac{1J}{erg.}$$

$J \rightarrow W = F \times x = kg\,m\,s^{-2} \times m.$

$= kg\,m^2 s^{-2}$

$[M^1 L^2 T^{-2}]$

$(1J)$

$n_2 u_2 = n_1 u_1$

$$n_2 = \left[\frac{m_1}{m_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$h = \left[\frac{1\,kg}{1\,g}\right]^1 \left[\frac{1\,m}{1\,cm}\right]^2 \left[\frac{1\,s}{1\,s}\right]^{-2}$$

$$h = \left[\frac{1000\,\cancel{g}}{1\,\cancel{g}}\right]^1 \left[\frac{100\,\cancel{cm}}{1\,\cancel{cm}}\right]^2 \left[\cancel{1s}\right]^{\frac{0}{1s}}$$

$$= [1000]^1 [100]^2 \qquad 10000 = 10^4$$

$$n = 10^3 \times 10^4$$

$$\boxed{1J = 10^7\,erg}$$

→ Derived the expresion for the time period (T) of a simple pendulam which may depend up on the mass (m) ~~smatten~~ of the body, length (l) of the pendulam and acc due to grovity (g)

$T \propto mlg$

$T \propto m l g$ $\qquad ms^{-2}$

$T \propto m^a l^b g^c$ $\qquad LT^{-2}$

$[M^0 L^0 T^1] \propto [M^1]^a [L]^b [LT^{-2}]^c$

$\propto [M^a] [L^b] [L^c T^{-2c}]$

$[M^0 L^0 T^1] \propto [M^0] [L^{b+c}] [T^{-2c}]$

$\boxed{a = a}$

$b + c = 0$

$b = -c$

$b = -(-1/2)$

$\boxed{b = +1/2}$

$-2c = 1$

$\boxed{c = \frac{-1}{2}}$

$T \propto m^0 L^{1/2} g^{-1/2}$

$g^{1/2} = \sqrt{g}$

$l^{1/2} = \sqrt{l}$

$T \propto 1 \ \frac{l^{1/2}}{g^{1/2}}$

$T \propto \frac{\sqrt{l}}{\sqrt{g}}$

$T \propto \sqrt{\frac{l}{g}} \Rightarrow T = k\sqrt{\frac{l}{g}}$

$\boxed{T = \propto 2\pi \sqrt{\frac{l}{g}}}$

→ The period T_0 of a planet of mass (m) above the sun in a cirqular orbit of radius (R) depends on (m), R G were G is the gravitasional constant point expresion for time period by dimenstional methods.

$$T_0 \rightarrow [M^0 L^0 T^1]$$

$$m \rightarrow \text{mass } [M]$$

$$R \rightarrow \text{Radius } [M^0 L^1 T^0]$$

$$G \rightarrow [M^{-1} L^3 T^{-2}]$$

$$T_0 \propto m^a R^b G^c$$

$$T_0 \propto [M]^a [M^0 L^1 T^0]^b [M^{-1} L^3 T^{-2}]^c$$

$$T_0 \propto [M^a][L^b][M^{-c} L^{3c} T^{-2c}]$$

$$T_0 \propto [M^{a-c}][L^{b+3c}][T^{-2c}]$$

$$[M^0 L^0 T^1] \propto [M^{a-c}][L^{b+3c}][T^{-2c}]$$

$a - c = a$ $\quad$ $b + 3c = 0$ $\quad$ $1 = -2c$

$\boxed{a = c}$ $\quad$ $b = -3c$ $\quad$ $c = -1/2$

$\boxed{a = -1/2}$ $\quad$ $= -3(-1/2)$ $\boxed{b = +3/2}$ $\quad$ $c = -1/2$

$T_0 \propto m^{-1/2} R^{+3/2} G^{-1/2}$

$T_0 \propto \frac{R^{3/2}}{m^{1/2} G^{1/2}}$

$T_0 \propto \frac{R^{3/2}}{\sqrt{m}\sqrt{G}}$

$$\boxed{T_0 \propto \frac{R^{3/2}}{\sqrt{mG}}}$$

If energy $[E]$ velocity (v) time $[T]$ of chosen as a fundoment quantitys. Find the dimentional formula of surface tenstion is defined as force for unit length

$\text{Surface} = \frac{F}{l} = kg \cancel{m} s^{-2}$

$= kg\, s^{-2}$

$= [MT^{-2}]$

$\therefore St \propto E^a v^b t^c$

$$\cancel{E = \frac{W}{t} = \frac{Fx}{t} = \frac{kgm^2s^{-2}}{s^{1}} = kgm^2s^{-3}}$$

$=$

$$E = \frac{1}{2}mv^2 = [ML^2T^{-2}]$$

$$= mgh$$

$$v = ms^{-1} = [M^0L^1T^{-1}]$$

$$T = S = [M^0L^0T^1]$$

$$S.T \propto [M^1L^2T^{-2}]^a [LT^{-1}]^b [T^1]$$

$$S.T \propto [M^aL^{2a}T^{-2a}][L^bT^{-b}][T^c]$$

$$S.T \propto [M^a][L^{2a+b}][T^{-2a-b+c}]$$

$$[M^1L^0T^{-2}] \propto [M^a][L^{2a+b}][T^{-2a-b+c}]$$

$\boxed{a=1}$

$$2a+b=0 \qquad -2=-2a-b+c$$

$$2(1)+b=0 \qquad -2=-\cancel{2}+\cancel{2})+c$$

$$\boxed{b=-2} \qquad \boxed{c=-2}$$

$$ST \propto E^1v^{-2}c^{-2}$$

$$\boxed{ST = \frac{E}{v^2c^2}}$$

~~Design~~ your own system of dimenstion and take velocity (v) plank constant = h $^{E \times t}$ & Gravitasional constant (G) as the fundamental ~~capital~~ quantities. What will be the dimentional of length mass (m) time (T) in the system.

~~E×t~~

Plank constant = h = E × t

$\boxed{E = h\nu}$ $\boxed{\nu = \frac{1}{T}}$

$E = \frac{h}{T}$ $\boxed{h = E \times t}$

$= [mL^2T^{-2}][T]$

$= h = [mL^2T^{-1}]$

Plankconstant $= \{ h = [m L^2 T^{-1}]$

$G = [M^{-1} L^3 T^{-2}]$

$h = [M^1 L^2 T^{-2}]$

$M, L, T \rightarrow$ dimentions.

$[M^1 L^1 T^1] = [\text{velocity}]^a [G]^b [h]^c$

$[M^1 L^1 T^1] = [L T^{-1}]^a [M^{-1} L^3 T^{-2}]^b [M^1 L^2 T^{-1}]^c$

$[M^1 L^1 T^1] = [L^a T^{-a}] [M^{-b} L^{3b} T^{-2b}] [M^c L^{2c} T^{-c}]$

$[M^1 L^1 T^1] = [M^{-b+c}] [L^{a+3b+2c}] [T^{-a-2b-c}]$

$-b + c = 1$

$\boxed{c = b}$

$a + 3b + 2c = 1$

$a + 3b + 2c = 1$

$a + 3b + 2b = 1$

$a + 5b = 1$

$a = 1 - 5b \Rightarrow a = 1 - 5(1)$

$\boxed{a = -4}$

$-a - 2b - c = 1$

$-(1 - 5b) - 2b - b = 1$

$-1 + 5b - 3b = 1$

$5b - 3b = 2$

$b = 2/2$

$\boxed{b = 1}$

$a = 1 - 5(1)$

$\boxed{a = -4}$

$[M^1 L^1 T^1] = v^{-4} G^1 h^1$

$\boxed{M^1 L^1 T^1 = \frac{G h}{v^4}}$

7 Suppuse your system of units has 1year has the unit of time, one light year as a unit of length and mass of the earth has the unit of mass. how much one nutan be = to the units of force in this system. take 1year = 3×10^5 sec, 1light year = 9.5×10^{15} m and mass of earth = 6×10^{24} kg.

$n_2 u_2 = n_1 u_1$ $\quad\quad n = 1$

$n_2 = n_1 \frac{u_1}{u_2}$ $\quad\quad N = kg\,m\,s^{-2}$

$= 1\left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

$n_2 = \left[\frac{1kg}{Me}\right]\left[\frac{1m}{1Ly}\right]\left[\frac{1s}{1y}\right]^{-2}$

$= \left[\frac{1kg}{6 \times 10^{24} kg}\right]\left[\frac{1m}{9.5 \times 10^{15} m}\right]\left[\frac{1s}{3 \times 10^5 s}\right]^{-2}$

$n_2 =$

→ Acuracy and Priccesion of instruments and errors in Measurement

→ errors

errors is the amount of uncertainity that is present in the measurement made with the measuring instrument.

* Types of errors.

i) Systamatic errors.

ii) Random errors.

→ Systamatic errors.

Error that is always unidiractional is called systamatic errors

Types of systamatic errors

i) instrumental errors

ii) Constant errors

iii) environmental errors

iv) Personal errors

1) Instrumental errors.

Instrumental errors that arrise from the errors due to imperfect ~~errors~~ disign or caliabration of instrument

→ Random errors.

The errors which are irragular who's cause is not ~~known~~ known, and random in nature in there sign, size are called random errors.

→ least count errors

The smallest value that can be measured by the measuring instrument is called least count.

→ Error(Δx):

A difference b/w measured value and true value

Ex: x = Measured value - ~~Total val~~ True value

Suppose true length of a body is 2.5cm
if it is measured as 2.7 cm, then the
error is +0.2

If it is measured as 2.2 cm, then the
error is -0.3 cm

→ Absolute error ;

The magnitude of the difference b/w
the true value of the measured physical
quantity and the value of individual
measurement is called the absolute
error.

$(\Delta x_i = x_i - x_{mean})$

$x_{mean} \rightarrow$ Actual value

$x_i \rightarrow$ nth observed value

(always positive)

→ Mean absolute error.

$$\Delta x_{mean} = \sum_{i=1}^{n} \left(\frac{\Delta x_i}{n} \right)$$

$$\Delta x_{mean} = \frac{(\Delta x_1)}{1} \quad n = 1$$

$$\Delta x_{mean} = \frac{(\Delta x_2)}{2} \quad n = 2$$

→ Relative error:-

$$\frac{\text{mean Absolute value}}{\text{true value}}$$

$$\text{Relative error} = \frac{\text{Absolute error}}{\text{Actual value}} = \frac{\Delta x_{mean}}{x_{mean}}$$

(There is no unit for relative error)

→ Percentage error = Relative error X 100

→ 2.23 example

In the measurement of a physical quantity $X = \frac{A^2 B}{C^{1/3} D^3}$. the percentage error introduced in the measurement of the quantities A, B, C and D are 2%, 2%, 4% and 5%, respectively then the minimum amount of percentage of error in the measurement of X is contributed by which quantity?

$$x = \frac{A^2 B}{3^{1/3} D^3}$$

Persentage error = Relative error × 100

Relative error = $\frac{\text{Mean absolute value}}{\text{true value}}$

$\frac{\Delta A}{A} = 2\%$. $\frac{\Delta C}{C} = 4\%$.

$\frac{\Delta B}{B} = 2\%$. $\frac{\Delta D}{D} = 5\%$.

$$\frac{\Delta x}{x} \times 100 = 2 \times \frac{\Delta A}{A} + 1 \times \frac{\Delta B}{B} + \frac{1}{3} \times \frac{\Delta C}{C} + 3 \times \frac{\Delta D}{D}$$

$$= (2 \times 2) + (1 \times 2) + (\tfrac{1}{3} \times 4) \times 3 \times 5.$$

$$= 4 + 2 + \frac{4}{3} + 15$$

$$= 21^{\times 3} \times \frac{4}{3}$$

$$= \frac{63 + 4}{3} = \frac{67}{3} = 22.34\%.$$

→ ~~Not~~ Note:-

i) In case of addition or subtraction ~~set~~

$$x \pm \Delta x = (A - B) \pm (\Delta A + \Delta B)$$

ii) In case of multiplication or division.

$$\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

iii) In case of power

If $Z = \frac{A^p B^q}{C^r}$

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

→ In an experiment of simple pendulum the error in measurement of length of the pendulum (L) and time perior are 3% and 2% respectively what is the maximum percentage of $\frac{L}{T^2}$?

$x = \frac{L}{T^2}$

$\frac{\Delta l}{L} = 3\%$

$\frac{\Delta T}{T} = 2\%$

$x = \frac{L}{T^2}$

$\frac{\Delta x}{x} = \left(1 \times \frac{\Delta L}{L}\right) + \left[2 \times \frac{\Delta T}{T}\right]$

$= (1 \times 3) + 2 \times 2$

$= 3 + 4$

$\frac{\Delta X}{X} = 7\%$

→ one side of cube is measured as $a = 4.03 \pm 1\%$

$V = a^3$

$= (4.03 \pm 1\%)^3$

$= (4.03)^3 \pm (3 \times 1)\%$

$= 64.27 \pm 3\%$

$\left(\frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}\right)$

$(1 + 1 + 1) = 3\%$

→ error in the measurements of radius of a sphere is one percent find the ~~error in the~~ measurement of volume.

$V = \frac{3}{4} \pi r^3$

$r = 1\%$

$\frac{\Delta V}{V} = \left(3 \times \frac{\Delta r}{r}\right)$

$\frac{\Delta V}{V} = 3 \times 1$

$\boxed{\frac{\Delta V}{V} = 3\%}$

$\Rightarrow$ significant rules.

i) All non zero digits are significant

Ex:- 487 → 3sf

0027 → 2sf

914875 → 6sf.

2) Zeros proceding (coming before) non zero digits are not significant

Ex:- 0.0037 → SF 2

0.387 → SF 3

3) Zeros in b/w the non zero digits are significant

Ex:- 9008 → 4SF

2.005 → 4SF

4) Zeros are significant right to the non zero digit when non zero digit is in decimal place.

0.200 → 3 SF

i) 0.007

ii) 2.64×10^{24}

iii) 0.2370.

iv) 6.320

v) 6.032

vi) 0.000603^2

5) For particular counting values we have infinity significant figures.

Ex:- 2 pens → 2.00000 . . .

20 Apples → 20.000 . . . ∞ SF

rounding of figures

i) 20.96

ii) 0.00031·25

iii) 6.7158

iv) 5728

→ dimentional formula for activity of a radioactive substance is.

→ $d = \frac{dR}{dt}$

$= \frac{1}{t}$

$= [M^0 L^0 T^{-1}]$

→ Check the aqurity for.

$\frac{1}{2} mv^2 = mgh$

$LHS = \frac{1}{2} mv^2$

$= kg\,(ms^{-1})^2$

$= kg\,m^2 s^{-2}$

$LHS = [M^1 L^2 T^{-2}]$

$RHS = mgh$

$= kg\,ms^{-2}\,m.$

$= kg\,m^2 s^{-2}$

$RHS = [M^1 L^2 T^{-2}]$

$LHS = RHS$

→ A physical quantity P is related to 4 quantities related to a, b, c and d as follows $P = \frac{a^3 b^2}{\sqrt{c}\, d}$ the percentage error of the measurements in a, b, c and d are 1%, 3%, 4%, 2% resepectively. What is the percentage error of the quantity P

→ $\frac{\Delta P}{P} = ?$, $\frac{\Delta a}{A} = 1\%$, $\frac{\Delta b}{b} = 3\%$, $\frac{\Delta c}{c} = 4\%$.

$\frac{\Delta d}{d} = 2\%$.

$$\frac{\Delta P}{P} = 3 \times \frac{\Delta a}{a} + 2 \times \frac{\Delta b}{b} + \frac{1}{2} \times \frac{\Delta c}{c} + \frac{\Delta d}{d}.$$

$$= 3 \times 1 + 2 \times 3 + \frac{1}{\cancel{2}} \times \cancel{4}^{2} + 2$$

$$= 3 + 6 + 2 + 2$$

$$= 13\%.$$

→ The measured mass and volume of a body or 2.42g & 4.7 cm^3 respectively with posible errors 0.01g and 0.1cc. Find the maximum errors in density

$$density = \frac{mass}{volume}$$

$$\frac{\Delta d}{d} = \left(\frac{\Delta m}{m} \times 1\right) + \left(\frac{\Delta V}{V} \times 1\right)$$

$$= \left(\frac{0.01}{2.42}\right) + \left(\frac{0.1}{4.7}\right)$$

$$\frac{\Delta d}{d} = 0.0221 \times 100$$

$$= 2.21\%$$

→ The heat generated in curcuit is dependent on the resistance, current and time of flow of electric current. If the error measured in the above or ~~1%, 2% and 1%~~ 1%, 2%, 1%, respectively. what is the maximum in heat.

$$H = \frac{i^2 R T}{J}$$

$$\cancel{H = \frac{i^2(2)(1)}{J}}$$

~~H~~ =

~~ΔH~~ $\frac{\Delta i}{i} = 2\%$

$\frac{\Delta R}{R} = 1\%$

$\frac{\Delta T}{T} = 1\%$

$$\frac{\Delta H}{H} = \left(2 \times \frac{\Delta i}{i}\right) + \left(1 \times \frac{\Delta R}{R}\right) + \left(\frac{\Delta T}{T}\right)$$

$$= (2 \times 2) + (1 \times 1) + (1 \times 1)$$

$$= (2 \times 2) + 1 + 1$$

$$= 4 + 1 + 1$$

$$\frac{\Delta H}{H} = 6\%$$

→ The velocity of a object veryes of time as $v = At^2 + Bt + C$. If units of v and t are expressed in SI find the units of the constant a, b and C.

$v = At^2$ $\quad v = Bt$ $\quad v = C$

$A = \frac{v}{t^2}$ $\quad B = \frac{v}{t}$ $\quad \boxed{C = ms^{-1}}$

$= \frac{ms^{-1}}{s^2}$ $\quad = \frac{ms^{-1}}{s}$

$\boxed{A = ms^{-3}}$ $\quad \boxed{B = ms^{-2}}$

→ useing dimentional methords, verify the correcteness of the following relation

i) $a = v/r^2$

$a \rightarrow$ acc^{-n}

$v \rightarrow$ velocity.

$r \rightarrow$ radius.

$a = \frac{v^2}{r}$ $v = (m s^{-1})^2$ $a = [M^0 L$

$r = s$

$\frac{m^2 s^{-2}}{m}$

$m^2 s^{-2} m^{-1}$

$m^1 s^{-2}$

$[M^0 L^1 T^{-2}]$

→ ~~vector~~

vectors

→ Physical quantity.

i) magnitude

ii) direction

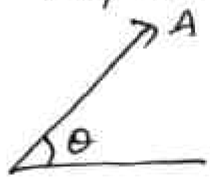
→ vectors :- vectors is defined as which has both magnitude and direction

Eg :- velocity, displacement, acceloration, etc

scalars :- Scalars are defined has which has only magnitude

Eg :- distance, speed, etc.

→ Geometrical Representation of vectors.



→ Types of vectors

i) Like vectors.

ii) unlike vectors.

iii) Zero vector.

iv) Axial vector.

v) Equal vector.

vi) Negative vator

vii) Parallel vector

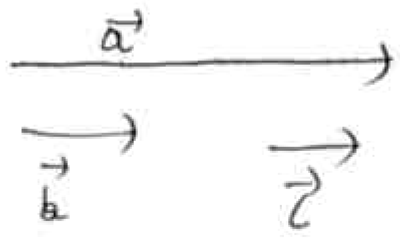
viii) Co-planar Vector

ix) Co-initial vector.

x) Unit vector

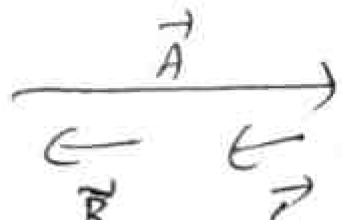
→ i) Like vectors:- Two or more vectors are having same physical quantity with same magnitude.

Ex:-



ii) Unlike vectors:- Two or more vectors having same magnitude but oppsite in direction are called unlike vectors.

Ex:-



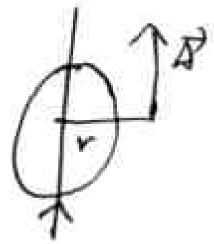
iii) zero vectors :- If ~~w~~ two vectors A and B are equal vectors, who's magnitude isequal to zero.

Ex :- $|\vec{A}| = 0$

$\vec{A} - \vec{B} = 0$

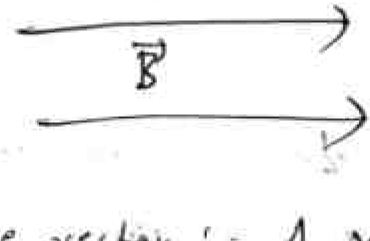
iv) Axial vector :- The vector having magnitude with Axial direction (circular path).

Ex :-



v) equal vectors :- two or more vectors are equal in magnitude and direction are called equal vector

Ex :-



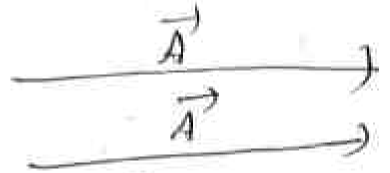
vi) Negitive vector :- A vector having the same magnetude and opposite in direction are called Negitive vector.

Ex :-

$\vec{A} = -\vec{B}$

vii) Parallel vector :- two or more vector are said to me parallel when they are act along ~~the~~ the some line

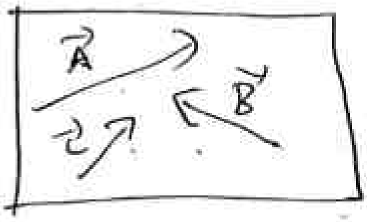
Ex:-



$\vec{A} = K\vec{A}$

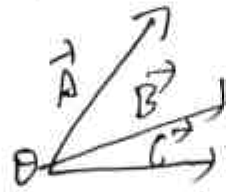
viii) Co-planer vector :- two or more vectors are present in a same plain with magnitude and direction

Ex:-



ix) Co-initial vector :- Who's vector orgenate from same point is colled co-inctial vector

Ex:-



x) Unit vector :- A vector who's magnitude is equal to one it is denoted as. $\{\hat{a}\}$

Ex:

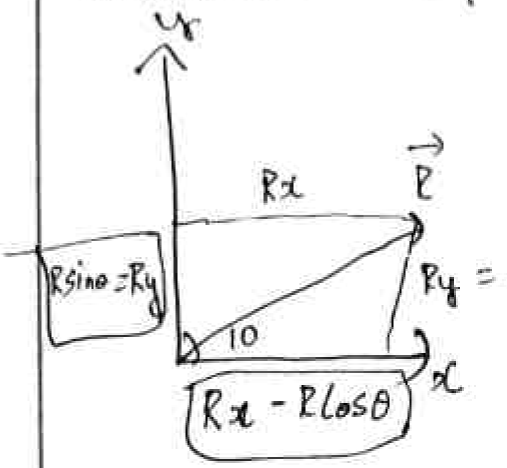
$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

If $a^{-1} = 1$

$|\vec{a}| = 1$

$\hat{a} = \frac{1}{1}$ $\hat{a} = 1$

→ Resolution of vectors



any vector quantity can be represented as sum of two or three vectors, which can be called as components of the vector.

along x-axis (x-components)

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos\theta = \frac{R_x}{\vec{R}}$$

$$\boxed{R_x = \vec{R}\cos\theta}$$

along y xis.

$$\sin\theta = \frac{opp\theta}{hyp}$$

$$\sin\theta = \frac{R_y}{\vec{R}}$$

$$\boxed{\vec{R}\sin\theta = R_y}$$

$$\frac{eq②}{eq①} = \frac{R_y = \vec{R}\sin\theta}{R_x = \vec{R}\cos\theta}$$

$$\tan\theta = \frac{R_y}{R_x}$$

magnitude are R_y & R_x

The direction along x & y axis

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

→ Properties of vectors.

i) Addition of vector

ii) Substraction of vector.

iii) Product of vector.

$$\vec{P} = \vec{A} - \vec{B}$$

$$\vec{A} - \vec{B} = -\hat{i} - \hat{j} - \hat{k}$$

$$\vec{P} = \vec{A} + \vec{B}$$

$$\vec{A} = 5\hat{i} + 2\hat{j} + \hat{k}$$

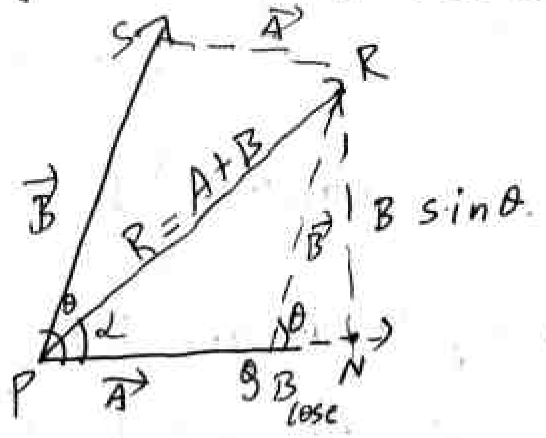
$$\vec{B} = 6i + 3\hat{j} + 2\hat{k}$$

$$\vec{A} + \vec{B} = 11\hat{i} + 5\hat{j} + 3\hat{k}$$

$$\vec{P} = 11\hat{i} + 5\hat{j} + 3\hat{k}$$

Note.

* Parallelogram law of vector addition



→ Let the two vector A & B, Inclianed at angle θ be acting on a partical at the same time. Let they be represented in magnitude and direction. by two adjcsent sides PQ & PS of llogram PQRS drawn from a point (P)

→ According llorgram law of vectors there resultent vector. R-vector will be represented by the diagnal PR. of the llogram

1) magnitude of the vector

$PQ = \vec{A}$ $\quad RN = \vec{B}\sin\theta$

$QN = \vec{B}\cos\theta$ $\quad PS = \vec{B}$

using pythogorus theorom

From ΔPRN

$PR^2 = PN^2 + RN^2$

$\vec{R}^2 = (PQ + QN)^2 + RN^2$

$= (\vec{A} + \vec{B}\cos\theta)^2 + (\vec{B}\sin\theta)^2$

$= \vec{A}^2 + \vec{B}^2(\cos^2\theta + \sin^2\theta) + 2\vec{A}\vec{B}\cos\theta$

$\vec{R}^2 = \vec{A}^2 + \vec{B}^2 + 2\vec{A}\vec{B}\cos\theta$

$\vec{R} = \sqrt{\vec{A}^2 + \vec{B}^2 + 2\vec{A}\vec{B}\cos\theta}$

Resultant of direction, α

$$\tan\alpha = \frac{RN}{PN} = \frac{RN}{PQ+QN} = \frac{B\sin\theta}{\vec{A} + \vec{B}\cos\theta}$$

Case - 1

→ If $\theta = 0^\circ$

$$\vec{R} = \sqrt{\vec{A}^2 + \vec{B}^2 + 2\vec{A}\vec{B}(1)}$$

$$\vec{R} = \sqrt{(\vec{A} + \vec{B})^2}$$

$$\boxed{\vec{R} = \vec{A} + \vec{B}}$$

case(2) → If $\theta = 180^\circ$

$$\vec{R} = \sqrt{\vec{A}^2 + \vec{B}^2 + 2\vec{A}\vec{B}(-1)}$$

$$= \sqrt{\vec{A} + \vec{B}^2 - 2\vec{A}\vec{B}}$$

$$= \sqrt{(\vec{A} - \vec{B})^2}$$

$$\boxed{\vec{R} = \vec{A} - \vec{B}}$$

case(3) → $\theta = 90$

$$\vec{R} = \sqrt{\vec{A}^2 + \vec{B}^2 + 2\vec{A}\vec{B}(0)}$$

$$\boxed{\vec{R} = \sqrt{\vec{A}^2 + \vec{B}^2}}$$

→ Resultant of direction

Case(I) ~~cos~~ $\theta = 0$

$$\tan\alpha = \frac{B\sin\theta}{\vec{A} + \vec{B}\cos\theta}$$

$$\tan\alpha = \frac{\vec{B}(0)}{\vec{A} + \vec{B}(1)}$$

$$\boxed{\tan\alpha = 0}$$

Case(ii) $\theta = 180$

$$\tan\alpha = \frac{\vec{B}(0)}{\vec{A} + \vec{B}(-1)} = 0$$

Case(iii) If $\theta = 90$

$$\tan\alpha = \frac{\vec{B}\sin(90)}{\vec{A} + \vec{B}\cos(90)}$$

$$= \frac{\vec{B}(1)}{\vec{A} + 0}$$

$$\boxed{\tan\alpha = \frac{B}{\vec{A}}} = \boxed{\tan\alpha = \frac{y}{x}}$$

$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$

or

$\alpha = \tan^{-1}\left(\frac{\vec{B}}{\vec{A}}\right)$

1) Find the sum of two vectors

$\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

$\cancel{\vec{A} + \vec{B} = 2\hat{i} + 3\hat{j} - 4\hat{k} + 2\hat{i} - 2\hat{j} + 4\hat{k}}$

$\cancel{\vec{B} + \vec{A} = 2\hat{i} - 2\hat{j} + 4\hat{k} + 2\hat{i} + 3\hat{j} - 4\hat{k}}$

$\vec{A} + \vec{B} = 4\hat{i} + \hat{j}$

$\vec{B} + \vec{A} = 4\hat{i} + \hat{j}$

$\vec{A} + \vec{B} = \vec{B} + \vec{A} \rightarrow$ commutative law

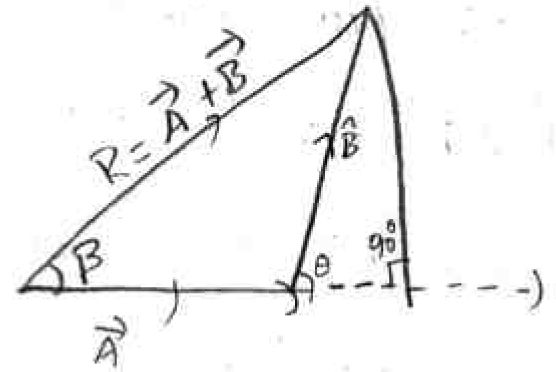
2) $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$

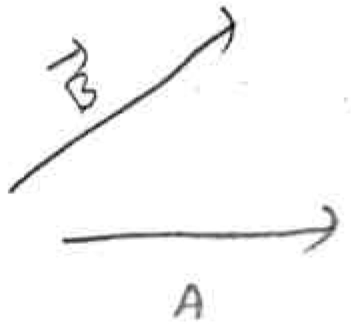
$\vec{B} = 2\hat{i} - \hat{j} - 2\hat{k}$

$\vec{A} + \vec{B} = 5\hat{i} - 3\hat{j} - \hat{k}$

$\vec{B} + \vec{A} =$

→ Triangular law of vector addition





→ Let the two vectors A & B inclined at an angle 'θ' be acting on a partical at the same time. let they represent magnetude and direction by two sides OP and PQ of a Δ OPQ taken in the same order.

Magnitude of resultant of two vectors by using pythagorus to ΔONQ

$$OQ^2 = ON^2 + QN^2$$
$$= (OP + PN)^2 + QN^2$$
$$= (\vec{A} + \vec{B}\cos\theta)^2 + (\vec{B}\sin\theta)^2$$

$\vec{R}^2 = \vec{A}^2 + \vec{B}^2 \cos^2\theta + 2\vec{A}\vec{B}\cos\theta + \vec{B}^2 \sin^2\theta$

$\vec{R}^2 = \vec{A}^2 + \vec{B}^2 (\cos^2\theta + \sin^2\theta) + 2\vec{A}\vec{B}\cos\theta$

$\vec{R}^2 = \vec{A}^2 + \vec{B}^2 + 2\vec{A}\vec{B}\cos\theta$

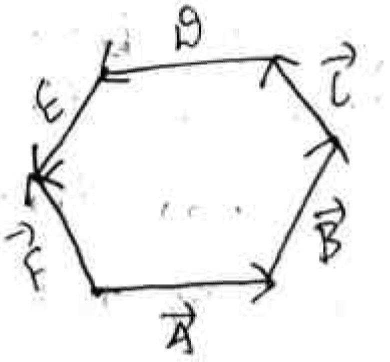
$\vec{R} = \sqrt{\vec{A}^2 + \vec{B}^2 + 2\vec{A}\vec{B}\cos\theta}$.

direction of the resultant, β

$$\tan\beta = \frac{\vec{B}\sin\theta}{\vec{A} + \vec{B}\cos\theta}$$

$$\beta = \tan^{-1}\left[\frac{\vec{B}\sin\theta}{\vec{A} + \vec{B}\cos\theta}\right]$$

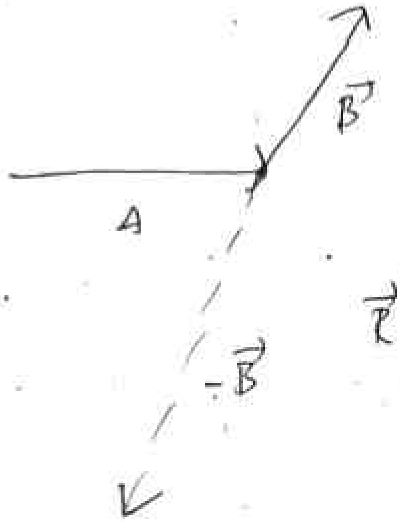
→ Polygon law of vector addition



$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} - \vec{F} = 0$

$$\boxed{\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} = \vec{F}}$$

* Subtraction of two vectors :-



$R = \vec{A} + \vec{B}$

$\vec{R} = \vec{A} + (-\vec{B})$

$= \vec{A} - \vec{B}$

Subtraction of two vectors defined as $\vec{A} + (-\vec{B})$ it will gives as a resultent

$R = A - B$

→ Find the subtraction of two vectors

i) $\vec{A} = 2\hat{i} - 3\hat{j} - 4\hat{k}$

$\vec{B} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{A} - \vec{B} = -2\hat{i} - \hat{j} - 6\hat{k}$

$\vec{B} - \vec{A} = 2\hat{i} + \hat{j} + 6\hat{k}$

$\vec{A} - \vec{B} = (2-4)\hat{i} + (-3-(-2))\hat{j} + (-4-(2))\hat{j}$

$= 2\hat{i} + (-3+2)\hat{j} + (-4-2)\hat{k}$

$= -2\hat{i} - \hat{j} - 6\hat{k}$

$\vec{B} - \vec{A} = (4-2)\hat{i} + (-2-(-3))\hat{j} + (2-(-4))\hat{k}$

$= 2\hat{i} + (-2+3)\hat{j} + (2+4)\hat{k}$

$\vec{B} - \vec{A} = 2\hat{i} + \hat{j} + 6\hat{k}$

$\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$ =

does not obeys commutative law

does not obeys Associative law.

substaction of two vectors.

i) $\vec{a} = 10\hat{j} + 6\hat{j}$

$\vec{b} = 4\hat{i} - 2\hat{j}$

$\vec{a} - \vec{b}$

ii) $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{b} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{b} - \vec{a}$

iii) $\vec{a} = 2\hat{i} - \hat{k}$

$\vec{b} = 3\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{a} - \vec{b}$

1) two non-zero vectors $\vec{A}$ & $\vec{B}$ are such that $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$. Find the angle b/w $\vec{A}$ & $\vec{B}$.

$\vec{R}^2 = \vec{R}^2$

$\cancel{A^2 + B^2} + 2AB\cos\theta = \cancel{\vec{A}^2 + \vec{B}^2} - 2AB\cos\theta$

$2AB\cos\theta + 2AB\cos\theta = 0$

$4AB\cos\theta = 0$

$\cos\theta = \frac{0}{4AB}$

$= \theta = \frac{1}{\cos}(0)$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$ $\theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}$ or 90

$\cos 90° = 0$

$90 = \cos^{-1}(0)$

$\cos\theta = 1$

$\theta = \cos^{-1}(1)$

2) If the sum of two unit vector is also a unit vector. Find the magnitude of there difference

→ $|\vec{A} - \vec{B}| = 2$

$|\vec{A} + \vec{B}| = 1$

$\vec{R}^2 = |A+B|^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2\vec{A}\vec{B}\cos\theta$

$1 = 1^2 + 1^2 + 2(1) \times (1) \cos\theta$

$1 = 2 + 2\cos\theta$

$1 - 2 = 2\cos\theta.$

$-1 = 2\cos\theta.$

$\boxed{\frac{-1}{2} = \cos\theta}$

$\vec{A} - \vec{B} = |\vec{A}|^2 + |\vec{B}|^2 - 2\vec{A}\vec{B}\cos\theta$

$= 1^2 + 1^2 - 2(1)\cos\theta$

$= 2 - 2\cos\theta$

$2\cos\theta = 2$

$\cos\theta = \frac{2}{2}$

$\boxed{\cos\theta = 1}$

$= 2 - 2\left(\frac{-1}{2}\right)$

$= 2 + \frac{2}{2}$

$|\vec{A} - \vec{B}|^2 = 3$

$\vec{R}^2 = 3$

$\vec{R} = \sqrt{3}$

Note:- $|\vec{A} + \vec{B}| = 1$

$|\vec{A} - \vec{B}| = 2$

$\vec{A} + \vec{B}$ $\vec{R} = \sqrt{\vec{A}^2 + \vec{B}^2 + 2\vec{A}\vec{B}} = \sqrt{|\vec{A} - \vec{B}|^2} = |\vec{A} + \vec{B}|$

$(\vec{A} - \vec{B})$ $\vec{R} = \sqrt{\vec{A}^2 + \vec{B}^2 - 2\vec{A}\vec{B}}$

$\vec{R}^2 = \vec{A}^2 + \vec{B}^2 + 2\vec{A}\vec{B}$

$\vec{R}^2 = \vec{A} + \vec{B} - 2\vec{A}\vec{B}$

→ The resultant of two forces who's magnitudes are in a ratio 3 : 5 is ~~20 i Newton~~ 28 newton. if the angle of there inclunation is $60°$ then find the magnitude of each forces.

→ F_1 & F_2 ; $\theta = 60$

3 : 5

$F_1 = 3x$

$F_2 = 5x$

$\vec{F} = 28N$

$$\vec{F} = \sqrt{\vec{F_1}^2 + \vec{F_2}^2 + 2\vec{F_1}\,\vec{F_2}\cos\theta\,\vec{R}} = \vec{F}$$

$$28 = \sqrt{(3x)^2 + (5x)^2 + 2 \times 3x \times 5x + \cos 60}$$

$$28 = \sqrt{9x^2 + 25x^2 + \overset{15}{\cancel{30}}x^2 \times \frac{1}{\cancel{2}}}$$

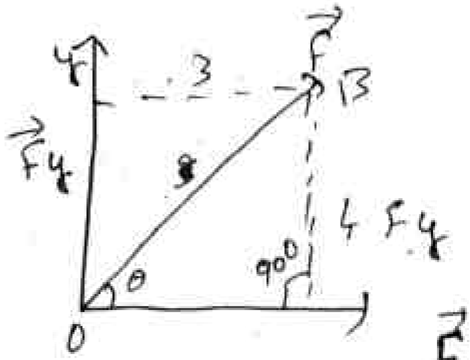
$$28 = \sqrt{9x^2 + 25x^2 + 15x^2}$$

$$28 = \sqrt{49x^2}$$

$$28 = 7x$$

$$\boxed{x = 4}$$

→ Resolution of a vector by the forcess



$$\vec{F_x}^{x} = \vec{F}\cos\theta$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos\theta = \frac{OA}{OB}$$

$$\cos\theta = \frac{\vec{Fx}}{F}$$

$$\vec{F_x} = \vec{F}\cos\theta$$

$\sin\theta = \frac{opp}{hyp}$

$\sin\theta = \frac{AB}{OB}$

$OB\sin\theta = AB$

$\boxed{\vec{F}\sin\theta = F_y}$

$\tan\theta = \frac{opp}{adj}$

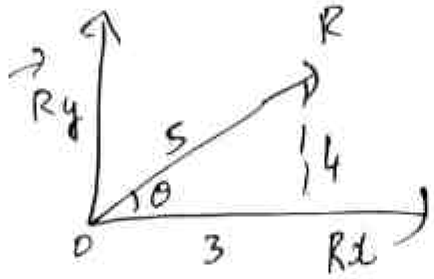
$\boxed{\tan\theta = \frac{4}{3}}$

$\theta = \tan\left(\frac{4}{3}\right)$

$\boxed{\theta = 53}$

Note:- $\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\vec{B} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$



→ Representation of vectors Rx and Ry along with x axis and y axis respectively the rectangular components of Rx are 3, Ry are 4, then

$$R^2 = \vec{R_x}^2 + \vec{R_y}^2 \qquad \vec{R_x} = 3$$

$$\vec{R_y} = 4$$

$$R = \sqrt{\vec{R_x}^2 + \vec{R_y}^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$\boxed{\vec{R} = 5}$$

it has maximum 53° in anti-clockwise direction with positive x-axis

Representation of in term of it's components.

We can write a convention that i, j & k be unit vector along x, y, z axis. A_x & A_y are ~~A~~ rectangular components in xy plain.

$$\vec{A} = \vec{A_x} + \vec{A_y}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{i}$$

$$\vec{A}^2 = \vec{A_x}^2 + \vec{A_y}^2$$

$$\vec{A} = \sqrt{A_x^2 + A_y^2}$$

$$\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$$

$$\vec{A} = \vec{A_x}\hat{i} + \vec{A_y}\hat{j} + A_z\hat{k}$$

$$\vec{A}^2 = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

→ direction cosines

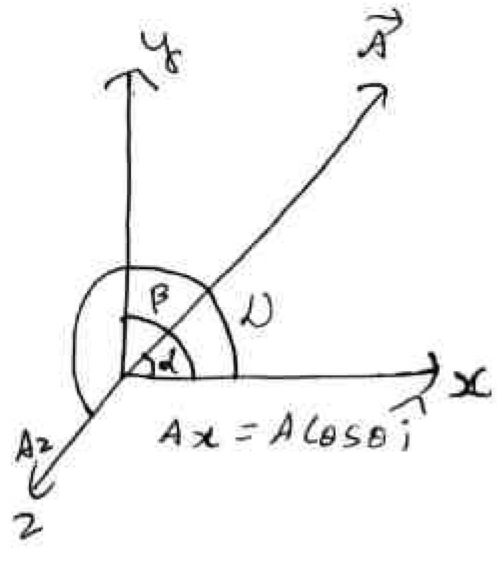
If α, β, γ are angle made by $\vec{A}$ with x, y, z axises ~~cause~~ cos α, ~~cause~~ cos β, ~~cose~~ cos γ are called direction cosines.

$$\vec{A}_x = \vec{A} \cos\theta$$

$$\cos\alpha = \frac{\vec{A}_x}{\vec{A}}$$

$$\cos\beta = \frac{\vec{A}_y}{\vec{A}}$$

$$\cos\gamma = \frac{\vec{A}_z}{\vec{A}}$$



$\vec{A} = \vec{A}_x + \vec{A}_y$

$\vec{A} = \vec{A}\cos\theta\,\hat{i} + \vec{A}\sin\theta\,\hat{j}$

$\vec{A} = \vec{A}\,(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

direction cosines

$\cos^2\alpha + \cos^2\beta + \cos^2 D = 1$

$LHS = \left(\frac{\vec{A}_x}{\vec{A}}\right)^2 + \left(\frac{\vec{A}_y}{\vec{A}}\right)^2 + \left(\frac{\vec{A}_z}{\vec{A}}\right)^2$

$= \frac{\vec{A}_x^{\,2}}{\vec{A}^2} + \frac{\vec{A}_y^{\,2}}{\vec{A}^2} + \frac{\vec{A}_z^{\,2}}{\vec{A}_z}$

$= \frac{1}{\vec{A}^2}\left[\vec{A}_x^{\,2} + \vec{A}_y^{\,2} + \vec{A}_z^{\,2}\right]$

$= \frac{1}{\cancel{\vec{A}^2}}\left[\cancel{\vec{A}^2}\right]$ $\vec{R} = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$\boxed{RHS = 1}$ $\therefore \vec{A} = \sqrt{\vec{A}_x^{\,2} + \vec{A}_y^{\,2} + \vec{A}_z^{\,2}}$

$\vec{A}^2 = \vec{A}_x^{\,2} + \vec{A}_y^{\,2} + \vec{A}_z^{\,2}$

$\vec{A} = \vec{A_x} + \vec{A_y}$

ii) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$LHS = (1-\cos^2\alpha) + 1(1-\cos^2\beta) + (1-\cos^2\gamma)$

$= +1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$

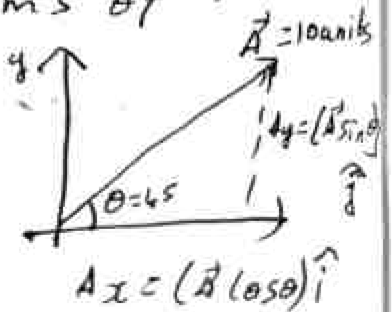
$= 3 - \cos^2\alpha - \cos^2\beta - \cos^2\gamma$

$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$= 3 - 1$

$= \underline{\underline{2}}$

iii) → A vector 'A' has magnitude 10 units & makes an angle 45° with x-axis in first ~~qodrant~~ quadrant, in terms of rectangular axis



→ $\vec{A} = \vec{A_x} + \vec{A_y}$

$= \vec{A}\cos\theta + \vec{A}\sin\theta$

$= 10\cos 45\hat{i} + 10\sin 45\hat{j}$

$= 10 \times \frac{1}{\sqrt{2}}\hat{i} + 10 \times \frac{1}{\sqrt{2}}\hat{j}$

$\vec{A} = \frac{10}{\sqrt{2}}\hat{i} + \frac{10}{\sqrt{2}}\hat{j}$

iv) If $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$

Find Magnitude and direction of these vector

$$\vec{A} + \vec{B} = 5\hat{i} + 7\hat{j}$$

Magnitude of the vector.

$$|\vec{A} + \vec{B}| = \sqrt{5^2 + 7^2}$$

$$= \sqrt{25 + 45}$$

$$|\vec{A} + \vec{B}| = \sqrt{74} \text{ units}$$

direction of the vectors.

$$\tan\theta = \frac{Ay}{Ax} = \frac{y}{x} = \frac{7}{5}$$

$$\theta = \tan^{-1}\left(\frac{7}{5}\right)$$

v) If vectors $\vec{A}$ & $\vec{B}$ all ~~$3\hat{i} + 4\hat{j}$~~
$\vec{A} = 3\hat{i} - 4\hat{j} + 5k$, $\vec{B} = 2\hat{i} + 3\hat{j} - 4k$, respectively
Find the unit vector // to ~~A+B~~ A+B

$$\hat{n} = \frac{\vec{A} + \vec{B}}{|\vec{A} + \vec{B}|}$$

$\vec{A}+\vec{B} = 5\hat{i}-\hat{j}+\hat{k}$

$|\vec{A}+\vec{B}| = \sqrt{5^2+(-1)^2+(1)^2}$

$= \sqrt{25+1+1}$

$= \sqrt{27}$

$$\hat{n} = \frac{\vec{A}+\vec{B}}{|\vec{A}+\vec{B}|} = \frac{5\hat{i}-\hat{j}+\hat{k}}{\sqrt{27}}$$

$$= \frac{5}{\sqrt{27}}\hat{i} - \frac{1}{\sqrt{27}}\hat{j} + \frac{1}{\sqrt{27}}\hat{k}$$

→ zero vector.

It is that vector which has zero magnitude. A zero vector is represented by $(\vec{0})$ It is also called as null vector.

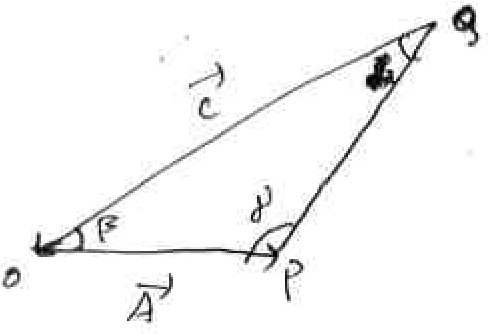
i) $\vec{A}\cdot(\vec{0}) = \vec{0}$

ii) $\vec{A}+(-\vec{A}) = \vec{0}$

Eg:- The velocity vector of a ~~stationary~~ partical.
stationary

The Accelaration vector of an object moving with uniform velocity is zerovector

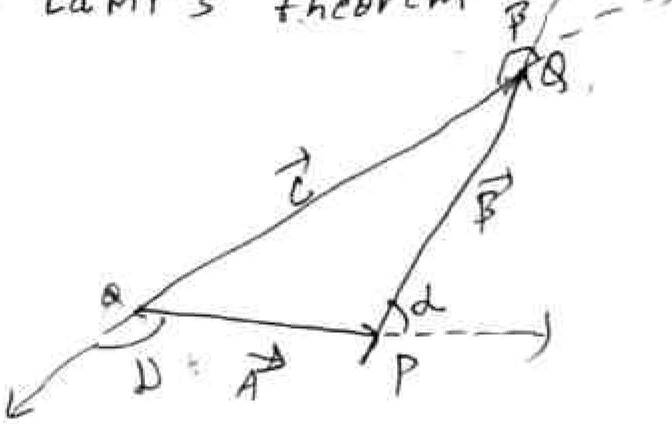
→ condition for zero resultant vector.

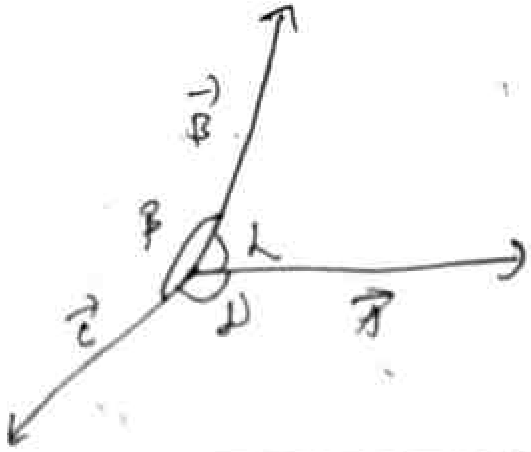


consider three vectors $\vec{A}, \vec{B}$ & $\vec{C}$ acting on an object at the some time let they me represented by $\vec{OP}, \vec{PQ}$ & $\vec{QO}$ that is the three sides of the Δ taken in one order it represented in magnitude and direction as the adjesent in sides of a closed polygon token in the same order, there resultant vector is zero.

$$\boxed{\frac{\vec{A}}{\vec{OP}} = \frac{\vec{B}}{\vec{PQ}} = \frac{\vec{C}}{\vec{QO}}}$$

→ ~~B~~ Lami's theorem





$$\boxed{\frac{\vec{A}}{\sin\alpha} = \frac{\vec{B}}{\sin\beta} = \frac{\vec{C}}{\sin\gamma}}$$

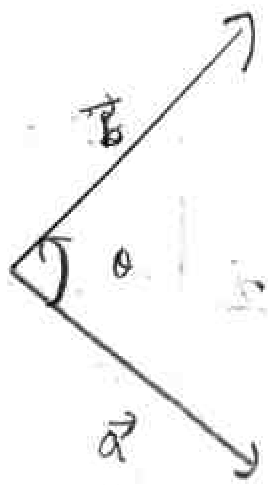
→ If three forcess acting at a same point for Equilibrium, then each force is propostional to the ~~sid~~ sin of angle b/w the other two forces.

Scalar product of two vectors.

(dot product)

The scalar product of two vector p & q is defined as the product of the magnitudes of p & q & a cosin of the angle θ formed by $\vec{P}$ ~~vector~~ & $\vec{Q}$ vector.

General formula



P & Q

$PQ = |\vec{P}|\,|\vec{Q}| \cos\theta$.

$\cos\theta = \dfrac{\vec{P}\,\vec{Q}}{|\vec{P}|\,|\vec{Q}|}$

$\theta = \cos^{-1} \dfrac{\vec{P}\,\vec{Q}}{|\vec{P}|\,|\vec{Q}|}$

$\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$

$i \cdot i = 1 \qquad i \cdot j = 0$

$j \cdot j = 1 \qquad j \cdot i = 0$

$k \cdot K = 1 \qquad K \cdot j = 0$

$\vec{a} \cdot \vec{b} = (4)(1) + 3(1) + 2(1)$

$= 4 + 3 + 2 \qquad \vec{a}\,\vec{b} = 9$

1) $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

$\vec{Q} = Q_x\hat{i} + Q_y\hat{j} + Q_z\hat{k}$

$PQ = (P_x Q_x) + (P_y Q_y) + (P_z Q_z)$

$\vec{P}\cdot\vec{P} = P_x^2 + P_y^2 + P_z^2$

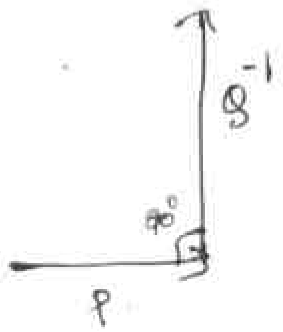
$\vec{P}^2 = P_x^2 + P_y^2 + P_z^2$

If $\vec{P} = \vec{Q}$

2) The 2 vector $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = A\hat{i} \mp 2\hat{j} - \hat{k}$ are perpendicular to each other. Find the value of A

$\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$

$\vec{Q} = a\hat{i} + 2\hat{j} - \hat{k}$



$PQ = |\vec{P}||\vec{Q}| \cancel{\cos} 90°$

$P \cdot Q = 0$

$(a\hat{i} + a\hat{j} + 3\hat{k})(a\hat{i} - 2\hat{j} - \hat{k}) = 0$

$a^2 + (-2a) + (-3) = 0$

$a^2 - 2a - 3 = 0$

3) $\vec{A} = 3\hat{i} + 4\hat{j}$

$\vec{B} = 12\hat{i} + 5\hat{j}$

Find the angle.

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

$|\vec{A}| = \sqrt{3^2+4^2} = 5$

$|\vec{B}| = \sqrt{12^2+5^2} = 13$

$$= \frac{(3\times12)+(5\times4)}{(5)(13)}$$

$$= \frac{36+20}{65}$$

$$\cos\theta = \frac{56}{65}$$

$$\theta = \cos^{-1}\left(\frac{56}{65}\right).$$

4) Calculate the work done if a partical displaces through $S = 2\hat{i} - \hat{j} + 5\hat{k}$ (m) ~~two~~ under the force $F = 4\hat{i} + 2\hat{j} - \hat{k}$ N

$S = 2\hat{i} - \hat{j} + 5\hat{k}$

$F = 4\hat{i} + 2\hat{j} - \hat{k}$

$W = F.S$

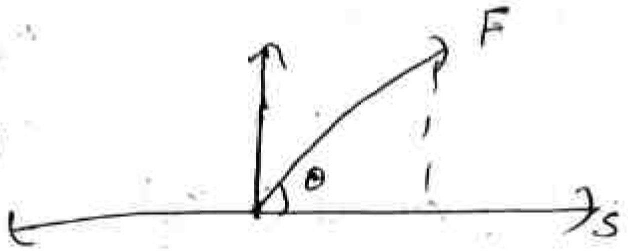
$= (2\hat{i} - \hat{j} + 5\hat{k})(4\hat{i} + 2\hat{j} - \hat{k})$

$= 1$ J

A. B = B. A → commutative law

A (B + C) = (A + B) C → distributive law

→ Examples of scalar products



$W = F \cdot S \cos\theta$

$F \cdot S = |\vec{F}| |\vec{S}| \cos\theta$

$W = F s \cos\theta.$

$W = Fs \cos(0)$ If $\theta = 0$

$\boxed{W = Fs}$ +ve

$W = F. S \cos(90)$

$\boxed{W = 0.}$ No work done

If $\theta = 190$

Cross vectors of

→ P & Q → General expresion.

$$P \times Q = |\vec{P}||\vec{Q}| \sin\theta \, \hat{n}$$

$$\sin\theta = \frac{P \times Q}{|\vec{P}||\vec{Q}|} \cdot \frac{1}{\hat{n}}$$

$$P = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$$

$$Q = Q_x\hat{i} + Q_y\hat{j} + Q_z\hat{k}$$

$$P \times Q = (P_x\hat{i} + P_y\hat{j} + P_z\hat{k}) \times (Q_z\hat{i} + Q_y\hat{j} + Q_z k)$$

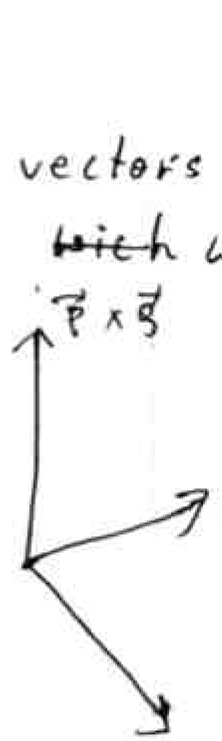
$$= \begin{vmatrix} i & j & k \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

$$= i \begin{vmatrix} P_y & P_z \\ Q_y & Q_z \end{vmatrix} + j \begin{vmatrix} P_x & P_z \\ Q_x & Q_z \end{vmatrix} + k \begin{vmatrix} P_x & P_y \\ Q_x & Q_y \end{vmatrix}$$

$$= i(P_yQ_z - P_zQ_y) + j(P_xQ_z - P_zQ_x) + k(P_zQ_y - P_yQ_x)$$

→ The cross product of two vectors P & Q is defined as the vector ~~wich~~ which obey the condition

① →



② $P \times Q = |\vec{P}||\vec{Q}| \sin\theta \, \hat{n}$

1) $(P\times Q) \neq (Q\times P)$

$P\times Q = -Q\times P$

$P\times(Q+R) = P\times Q + P\times R$

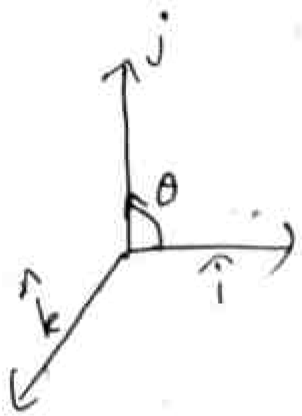
→ vector product Represented in Rectangular components

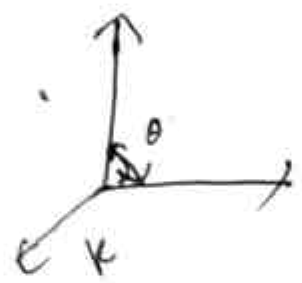
$i\times j = k$ $\quad$ $i\times k = -j$

$j\times k = i$ $\quad$ $k\times j = -i$

$k\times i = j$ $\quad$ $j\times i = -k$

$j\times j = 0$ $\quad$ $k\times k = 0$ $\quad$ $i\times i = 0$





1) Two vector $\vec{A}$ & $\vec{B}$ are inclined to each other at an angle 'θ'. Find a unit vector which is perpendicular to both 'a ~~and~~ & 'b',

$\vec{A}$ & $\vec{B}$

$\hat{n}$

$$\vec{A} \times \vec{B} = \vec{A} . \vec{B} \sin\theta \hat{n}$$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{\vec{A} . \vec{B} . (\sin 90)}$$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A}| \, |\vec{B}|}$$

2) Find a cross b if $\vec{a} = \hat{i} - 2\hat{j} + 4\hat{k}$ & $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 1 & -2 & 4 \\ 3 & -1 & 2 \end{vmatrix} = i\left(-4-(-4)\right) + j(2-12) + k\left(-1-(-6)\right)$$

$$= i(-4 \cancel{+} 4)^{0} + j(+10) + k(+5)$$

$$= 0\hat{i} + 10\hat{j} + 5\underline{\underline{k}}$$

$$= +10\hat{j} + 5\underline{\underline{k}}$$

3) $\vec{A} = (2\hat{i} + \hat{j})$

$\vec{B} = (\hat{i} - \hat{j} + 5\hat{k})$

$\vec{A} \times \vec{B}$

4) $\vec{A} \times B$

$\vec{A} = 2\hat{i} - \hat{j} + 5\hat{k}$

$\vec{B} = 4\hat{i} + 2\hat{j} - \hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & -j & k \\ 2 & -1 & 5 \\ 4 & 2 & -1 \end{vmatrix} = i \begin{vmatrix} -1 & 5 \\ 2 & -1 \end{vmatrix} - j \begin{vmatrix} 2 & 5 \\ 4 & -1 \end{vmatrix} + k$$

$$\begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix}$$

$$= i(+1 - 10) - j(2 - 20) + k(4 - (-4))$$

$$= -9\hat{i} - \hat{j}(-22) + 8\hat{k}$$

$$= -9\hat{i} + 22\hat{j} + 8\hat{k}$$

Ex: 3.14

$\vec{A} \times B$

$\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{B} = \hat{i} - \hat{j} + 6\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 1 & -1 & 6 \end{vmatrix}$$

$$= i \begin{vmatrix} 3 & -1 \\ -1 & 6 \end{vmatrix} - j \begin{vmatrix} 2 & -1 \\ 1 & 6 \end{vmatrix} + k \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$$

$$= i(18-1) - j(12+1) + k(-2-3)$$

$$= 17\hat{i} - 13\hat{j} - 5k$$

$$\vec{B} \times \vec{A} = \begin{vmatrix} i & -j & k \\ 1 & -1 & 6 \\ 2 & 3 & -1 \end{vmatrix}$$

$$\vec{B} \times \vec{A} = -17\hat{i} + 13\hat{j} + 5\hat{k}$$

$$\sin\theta = \frac{(\vec{B} \times \vec{A})}{(\vec{B}|(\vec{A})} = \frac{-17\hat{i} + 13\hat{j} + 5k}{\sqrt{38}\ \sqrt{14}}$$

$$|\vec{A}| = \sqrt{2^2 + 3^2 + (-1)^2} \qquad |\vec{B}| = \sqrt{1^2 + (-1)^2 + 6^2}$$

$$= \sqrt{4 \times 9 \cancel{+1}} \qquad = \sqrt{1+1+36}$$

$$= \sqrt{14} \qquad = \sqrt{38}$$

$$|B \times A| = \sqrt{17^2 + 13^2 + 5^2}$$

$$= \sqrt{289 + 169 + 25}$$

$$= \sqrt{483}$$

$$\sin\theta = \frac{\sqrt{483}}{\sqrt{38}\ \sqrt{14}}$$

$$\theta = \sin^{-1}\left(\frac{\sqrt{483}}{\sqrt{38}\sqrt{14}}\right)$$

$$= \sin^{-1}\left(\sqrt{\frac{483}{532}}\right)$$

→ Motion in a moving objects
Motion in a stright line (kinematics)

* kinematics deales with the study of distance, displacement, speed, velocity Accelaro~~tiom~~tion and time

* Rest ~~an~~ or motion.

→ Rest: If the object is in ~~start~~ static position or the object or not changeing its place from one place to another

Eg: Book lie on the table
→ Parking of a car
→ Board on the wall

→ Motion: Motion is also called a dinomic state of an object or the object ic changing it position from inetial place to final place.

Eg: → A car moving.
→ childrens are continasely playing
→ second hand of the watch.
→ Rotation of fan.

IMP * Distance or Displacement

→ Distance.

The actual length of the path covered by A moving partican in b/w two points is called distance of a object a distance is denoted as SI unit is 'Meter'

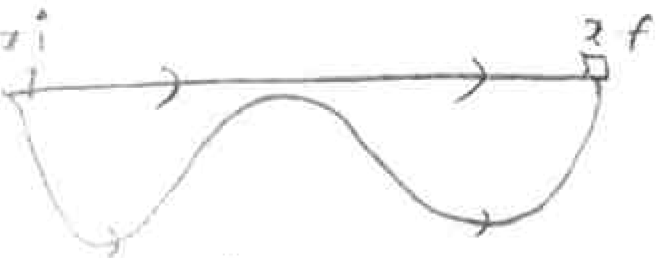
→ It has only magnitude

→ It is a scalar quantity

Ex:-

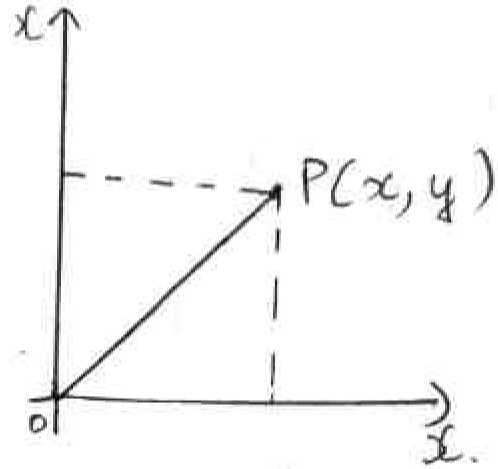
→ Displacement: Displacement is defined as an object is traveling in shortest path from inetial to final ~~pof~~ position.

Ex:-

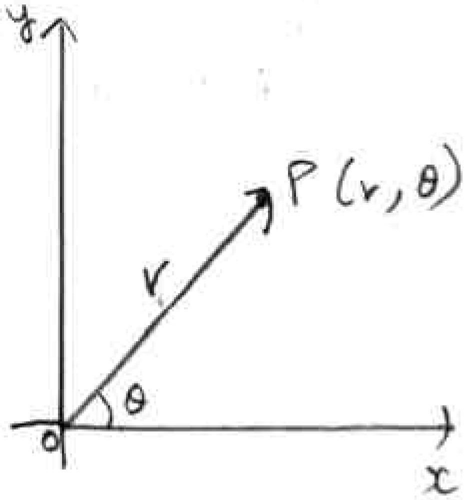


It is denoted as 'x'
SI unit of displacement is meter and it is avector quantity
displacement has both magnitude & direction.

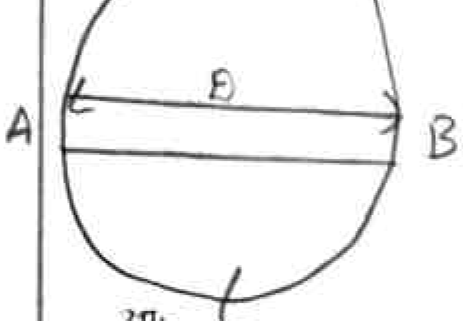
~~Cartition~~ cartesian co-ordinate system.

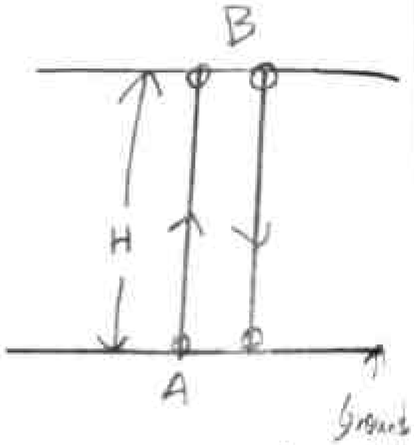


→ Polar co-ordinates



→ Reprasentation of distance and displacement





$\frac{2\pi r}{2} = \pi r$

$A \to B = \pi r$

A to A → distane $= 2\pi r$
displacement $= 0$

A to B → distance $= \pi r$
displacement $= D = 2r$

At A → distance → [illegible]
displacement $= 0$

A to B → distance → h
→ displacement → h

* Speed and velocity

→ speed

The distonce ~~covet~~ covered by a body in a unit time is called speed.

$$speed = \frac{s}{t}$$

SI unib of speed = ~~m/per~~ m/s

speed is a scalar quantity.

dimentional formula of speed = $[LT^{-1}]$

~~Avegage~~ ~~Avg speed = The total speed distant.~~

$$Avg\ speed = \frac{\text{The total distance travled by the object}}{\text{The total time interval.}}$$

If a partical travels the distonce s in time t_1 and t_2 the avg speed is

~~Avegar~~ $$Average\ speed = \frac{\text{total distance}}{\text{total time}} = \frac{s}{(t_1+t_2)}$$

→ Instantoneous speed

The Avg speed or the change in speed called Instantaneous speed.

(or)

The Avg speed Δs with-respect to change in time that is ~~ΔD~~ Δt

Instancous speed :- $Speed = time \frac{\Delta s}{\Delta s}$ $t \to 0$.

$$Inst\ speed = \frac{ds}{dt}$$

JEE mains

Avg speed in difference gercumstances

$$\text{Avg Speed} = \frac{\text{total distance covered}}{\text{total time taken.}}$$

$$\text{Avg speed} = \left(\frac{s_1 + s_2 + s_3 + \dots}{t_1 + t_2 + t_3 + \dots}\right)$$

Case 1.

~~Bod~~ two Bodys moving with different distances with different speed.

$$V_{avg} = \frac{s_1 + s_2 + s_3 + \dots}{t_1 + t_2 + t_3 + \dots}$$

v = velocity

$$= \frac{s_1 + s_2 + s_3 + \dots}{\left(\frac{s_1}{v_1}\right) + \left(\frac{s_2}{v_2}\right) + \left(\frac{s_3}{v_3}\right) + \dots}$$

$v = \frac{s}{t}$

$t = \frac{s}{v}$

$t_1 = \frac{s_1}{v_1}$

$t_2 = \frac{s_2}{v_2}$

$t_3 = \frac{s_3}{v_3}$

for two bodies

$$V_{avg} = \frac{s_1 + s_2}{\left(\frac{s_1}{v_1}\right) + \left(\frac{s_2}{v_2}\right)}$$

$$\boxed{V_{avg} = \frac{s_1 + s_2}{s_1 v_2 + s_2 v_1}}$$

ii) Case (ii)

$S_1 = S_2 = S$

$$v_{avg} = \frac{S_1 + S_2}{\left(\frac{S_1}{v_1}\right) + \left(\frac{S_2}{v_2}\right)}$$

$$= \frac{S + S}{\left(\frac{S}{v_1} + \frac{S}{v_2}\right)}$$

$$\text{avg Speed} = \frac{2\cancel{S}}{\cancel{S}\left(\frac{1}{v_1} + \frac{1}{v_2}\right)} = \frac{2}{\frac{v_2 + v_1}{v_1 v_2}} = \frac{2 v_1 v_2}{v_1 + v_2}.$$

iii) Two bodies moving with diffrent speeds in different time intervals.

$$v_{avg} :- \frac{S_1 + S_2}{t_1 + t_2}$$

$$= \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \dots}{t_1 + t_2 + t_3 + \dots}$$

$S = ?$

$v = \frac{s}{t}$

$s = vt$

$s_1 = v_1 t_1$

$s_2 = v_2 t_2$

$s_3 = v_3 t_3$

for two bodies

If $t_1 = t_2 = t$

$$= \frac{V_1 t_1 + V_2 t_2}{t_1 + t_2}$$

$$= \frac{V_1 t + V_2 t}{t_1 + t_2}$$

$$= \frac{V_1 t + V_2 t}{t + t}$$

$$= \cancel{t}\left(\frac{V_1 + V_2}{2\cancel{t}}\right)$$

$$V_{avg} = \frac{V_1 + V_2}{2}$$

iv) A motor vehicle traveled first $1/3$ of a distance s at a speed of $V_1 = 10$ kmph the secound $1/3$ at a speed $V_2 = 20$ kmph and the last $1/3$ at a speed of $V_3 = 10$ kmph. Determine the avg speed of the vehicle over a entire a distonse 's'

1st 1/3 2nd 1/3 3rd 1/3

$V_1 = 10\,kmph$ $V_2 = 20\,kmph$ $V_3 = 10\,kmph$

s_1 s_2 s_3

$s \rightarrow$ total distance.

$$V_{avg} = \frac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3}$$

$$= \frac{s_1 + s_1 + s_1}{\frac{s_1}{V_1} + \frac{s_2}{V_2} + \frac{s_3}{V_3}}$$

$$= \frac{s + s + s}{\frac{s_1}{V_1} + \frac{s}{V_2} + \frac{s}{V_3}}$$

$$V_{avg} = \frac{3s}{s\left(\frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3}\right)}$$

$s_1 = s$

$s_2 = s$

$s_3 = s$

$$V = \frac{s}{t}$$

$$t_1 = \frac{s_1}{V_1} = \frac{s}{V_1}$$

$$V_{avg} = \frac{3}{\frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3}}$$

$$V_{avg} = \frac{1}{\frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3}}$$

$$\frac{3}{V_{avg}} = \frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3}$$

$$\frac{3}{V_{avg}} = \frac{1}{10} + \frac{1}{20} + \frac{1}{10}$$

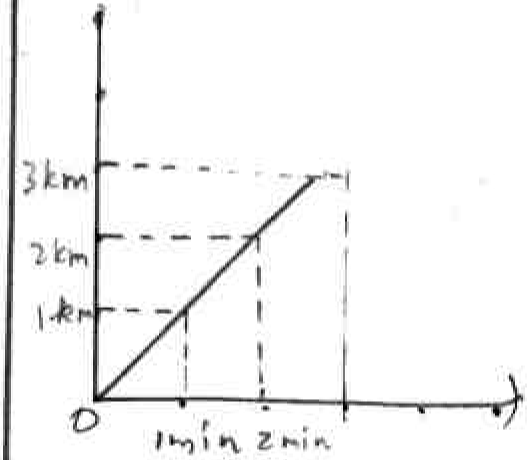
$$= 0.1 + 0.05 + 0.1$$

$$\frac{3}{V_{avg}}$$

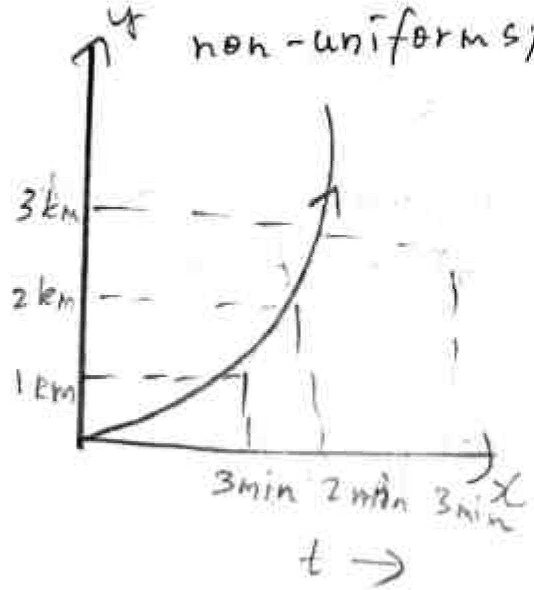
$$V_{avg} = \frac{3}{0.25} = 12\ kmph$$

* Uniform speed and non-uniform

→ Uniform speed.



→ non-uniform speed



$$\text{velocity} = \frac{\text{displacement}}{\text{time}} = \frac{x}{t}$$

$$\text{Avg velocity} = \frac{\text{total displacement}}{\text{total time take}}$$

$$= \frac{x_1 + x_2 + x_3 + \cdots}{t_1 + t_2 + t_3 + \cdots}$$

* Instanceous velocity :- $\lim \frac{\Delta x}{\Delta t}$

$\boxed{V_{Inst} = \frac{dx}{dt}}$

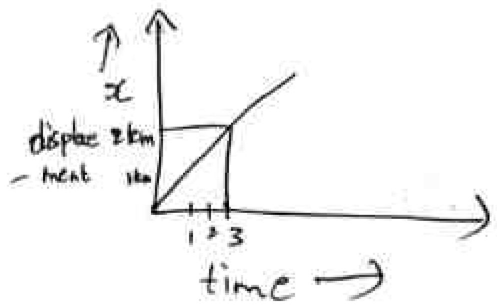
velocity :- It is defined as displacement of an object with respective time

SI unit of velocity = m/s

Dimentional formula = $[M^0 L^1 T^{-1}]$

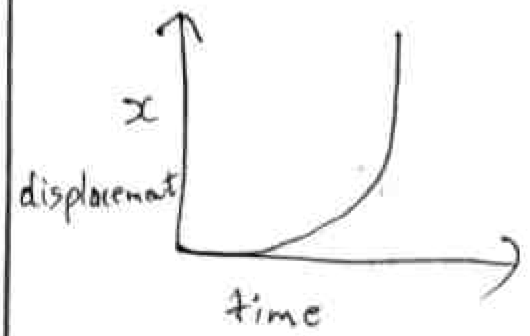
It is the vector quantity.

Uniform ~~Average~~ velocity :



When a partical covers equal displacement in equal interval of time

non-uniform velocity



When a body covers equal displacement in an equal intervol of time.

* Acceleration :

The change in velocity in unit time is called acceleration.

$$\frac{\Delta v}{t} \quad ms^{-2}$$

SI unit of acceleration = m/s^{-2}
Dimintional formula = $[M^0 L^1 T^{-2}]$

~~avg acc~~

→ Average acceleration.

It is defined as change in velocity by change in time.

$$\text{avg acc} = \frac{\text{change in velocity}}{\text{Change in time.}}$$

$$\vec{a}_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t}$$

→ Instaneous ~~Instantanies~~ acceleration.

$$= \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$\vec{a} = \frac{dv}{dt} \Rightarrow a = \frac{dv}{dt} \times \frac{dx}{dx}$$

$$a = \frac{dv}{dx} \times \frac{dx}{dt}$$

$$\boxed{a = v\frac{dv}{}}$$

* deriving equations of motion using calculur methoud

Let conseder an object moves in astight line withe uniform acc 'a'. The inetial velocity of the object is small 'u' $(t=0)$ and 'v' is its velocity after time interval t $[t=t]$

velocity - time.

$$\vec{a} = \frac{dv}{dt} = \frac{\text{Changevelodty}}{\text{time}}$$

$\vec{a} \cdot dt = d\vec{v} \rightarrow (1)$

Integrate eq(1) w.r.t 't' with the limits.

from $0 \rightarrow t$ and $u \rightarrow v$

$$\int_u^v dv = \int \vec{a} \cdot dt$$

$$\boxed{v = u + at}$$

$$[v]_u^v = \vec{a} \int_0^t dt$$

$v - u = a[t]_0^t$

$v - u = a[t - 0]$

$v - u = at.$

* Distance vs time.

Let us consider an object moves in a stright line with uniform acc 'a'. ~~let us cosider an~~ at any instant 't' the distance traveled by the object time interval Δt is Δs. then the instantainous velocity of the object is given by.

2) velocity - distance.

$$v = \frac{ds}{dt}$$

$$v\,dt = ds$$

$$ds = v.dt$$

Integrate on both sides w.r.t 't'

$0 \to s$ distance $0 \to t$ time.

$$\int ds = \int (u + at)\,dt \quad \Bigg| \quad \int t.dt = \frac{t^2}{2}$$

$$\int_0^s ds = \int_0^t u.dt + \int_0^t at.dt$$

$$[s]_0^2 = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

$$s - 0 = u(t-0) + a\left[\frac{t^2}{2} - \frac{0^2}{2}\right]$$

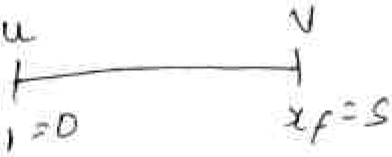
$$s = ut + \frac{at^2}{2}$$

$$\boxed{s = ut + \tfrac{1}{2}at^2}$$

→ velocity ~~dim~~ displasement relation.

3) $a = \frac{dv}{dt} = \frac{dv}{dt} \times \frac{ds}{ds}$ { Multiplys ÷ by ds.

$a = \frac{dv}{ds} \frac{ds}{dt}$



$a = v.\frac{dv}{ds}$

$a.ds = v.dv \rightarrow (1)$

Integrate eq in both sides

limits $0 \rightarrow s$ $u \rightarrow v$

displacement velocity.

$$\int_0^s a.ds = \int_u v.dv$$

$$a[s]_0^s = \left[\frac{v^2}{2}\right]_a^v$$

$\int v.dv = \frac{v^{n+1}}{n+1}$

$= \frac{v^2}{2}$

$$a[s-0] = \left[\frac{v^2}{2} - \frac{u^2}{2}\right]$$

$$as = \frac{1}{2}\left[v^2 - u^2\right]$$

$$\boxed{2as = v^2 - u^2}$$

A bullet moving with speed 150 m/s striks a tree and penintrates 3.5 cm before stoping. Find the ~~acc~~ magnitude of its acc and the time take to stop

Given

From 3^{rd} eq^n

$$v^2 - u^2 = 2as.$$

$$0^2 - (150)^2 = 2 \times a \times 0.035$$

$$-22500 = 0.07a$$

$$a = \frac{-22500}{0.07}$$

$$a = -321428.57$$

$$a = -3.21 \times 10^5 \, m/s$$

$$v = u + at$$

$$\boxed{t = \frac{v - u}{a}}$$

$$0 = 150 + 3.21 \times 10^5 \times t$$

$$t = \frac{0 - 150}{3.21 \times 10^5}$$

$$= \frac{-150}{-3.21 \times 10^5}$$

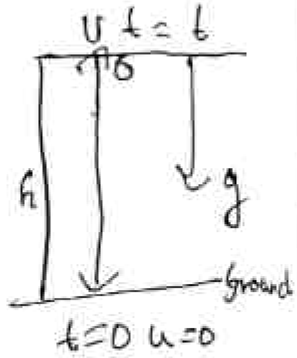
$$t = 46.72 \times 10^{-5} s.$$

* Acceleration due to Gravity (Motion under Gravity)

~~$v = u + at$~~

let an object (ball) moving in upword direction (oppesing of acc - due to gravity)

ball is in up word direction.



1) $v = u + (-g)t$ $\qquad a = -g$

$$\boxed{v = u - gt}$$

2) $s = ut + \frac{1}{2}at^2$ $\qquad \left\{ \begin{array}{l} a = -g \\ s = h \end{array} \right.$

$h = ut + \frac{1}{2}(-g)t^2$

$$\boxed{h = ut - \frac{1}{2}gt^2}$$

3) $v^2 - u^2 = 2as$ $\qquad \begin{array}{l} a = -g \\ s = h \end{array}$

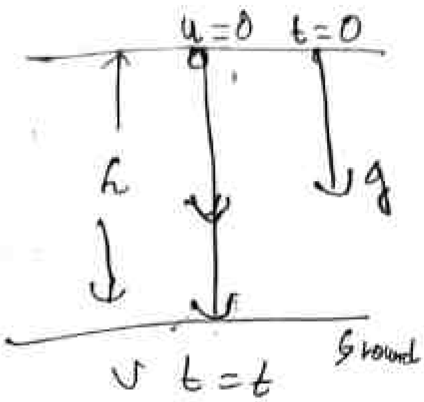
$v^2 - u^2 = 2 \times (-g)h$

$$\boxed{v^2 - u^2 = -2gh}$$

ball is in down direction.

1) $v = u + at$

$\boxed{v = u + gt}$ $a = g$



2) $S = ut + \frac{1}{2}at^2$

$\boxed{h = ut + \frac{1}{2}gt^2}$ $a = g$, $S = h$.

3) $v^2 - u^2 = 2as$

$S = h$

$\boxed{v^2 - u^2 = 2gh}$ $a = g$.

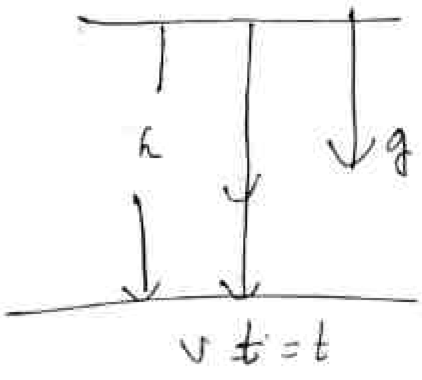
(or)

1) $v = u + at$ $a = g$

$v = 0 + gt$ $u = 0$

$\boxed{v = gt}$ $v = v$

$t = t$



2) $S = ut + \frac{1}{2}at^2$

$h = 0 \times t + \frac{1}{2}gt^2$

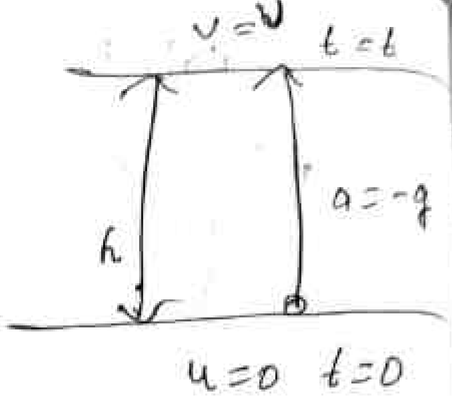
$h = \frac{1}{2}gt^2$

3) $v^2 - u^2 = 2as$

$v^2 = 2gh$

$v = \sqrt{2gh}$

2) upword direction.



1) $v = u + at$

$v = 0 + (-g)t$

$\boxed{v = -gt}$

$a = -g$
$u = 0$
$v = v$
$t = t$

2) $s = ut + \frac{1}{2}at^2$

$h = 0 \times t + \frac{1}{2}(-g)t^2$

$\boxed{h = -\frac{1}{2}gt^2}$

$s = h$
$u = 0$
$a = -g$
$t = t$

3) $v^2 - u^2 = 2as$

$v^2 - 0^2 = 2 \times (-g)h$

$v^2 = -2gh$

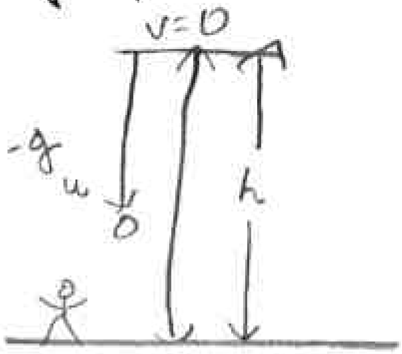
$\boxed{v = \sqrt{-2gh}}$

* Equation's of motion of an object vertically projected from the ground.

→ If the body projected vertically upwards, then conseder acc due to gravity of a free falling body as positive we take a negitive value for a body projected upwards

* Maximum height

When it is projected vertically upwards with an velocity 'u'



$v^2 - u^2 = 2a.$

$0^2 - u^2 = 2(-g) h_{max}$

$\cancel{-}u^2 = \cancel{-}2g h_{max}$

$\frac{u^2}{2g} = h_{max}$

* Time of ascent :-

Inetial velocity ~~u~~ u meter/s

acc of body = $-g$ m/s

Final velocity = 0

$t = ?$ ~~$v = u + at$~~ ~~$0^2 - u^2 = 2(-g)h_{max}$~~

$v = u + at$

$0 = u - gt$

$gt = u$

$\boxed{t_a = \frac{u}{g}}$

→ Time of decent.

~~Initial velocity~~

$S = ut + \frac{1}{2}at^2$ $\quad t = t_d$

$h_{max} = 0 \times t + \frac{1}{2}(+g)t^2$

$h_{max} = \frac{1}{2}g\,t_d^2$

$t_d^2 = \frac{2h_{max}}{g}$

$t_d = \sqrt{\frac{2h_{max}}{g}}$

$= \sqrt{\frac{\cancel{2}}{g} \times \frac{u^2}{\cancel{2}g}}$

$= \sqrt{\frac{u^2}{g^2}}$

$t_d = \frac{u}{g}.$

Total time of flight $= t_a + t_d$

$= \frac{u}{g} + \frac{u}{g}$

$T_f = \frac{2u}{g}.$

→ velocity ~~at~~ a given height

vertically upword direction.

Let a body be projected vertically upwords with an inetial velocity 'u' from the ground.

$v^2 - u^2 = 2as$ $\quad s = h$

$v^2 - u^2 = 2(-g)h$ $\quad a = -g$

$v^2 = u^2 - 2gh$

$v = \pm\sqrt{u^2 - 2gh}.$

vertically down direction.

$V = \pm\sqrt{u^2 + 2gh}.$

→ Time taken to ~~attione~~ attime a given height

$s = ut + \frac{1}{2}at^2$

$h = ut - \frac{1}{2}gt^2$ $\quad a = -g$, $s = h.$

$\frac{1}{2}gt^2 - ut + h = 0$

$\boxed{gt^2 - 2ut + 2h = 0}$

$ax^2 + bx + c = 0$

$a = g$ $b = -2u$, $C = 2h.$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{+2u \pm \sqrt{4u^2 - 4 \times g\,2h}}{2 \times g}$

$t = \frac{2u \pm \sqrt{4u^2 - 8gh}}{2g}.$

$$\boxed{t = \frac{2u \pm \sqrt{4u^2 - 8gh}}{2g}}$$

$$t = \frac{2u \pm \sqrt{4(u^2 - 2gh}}{2g}$$

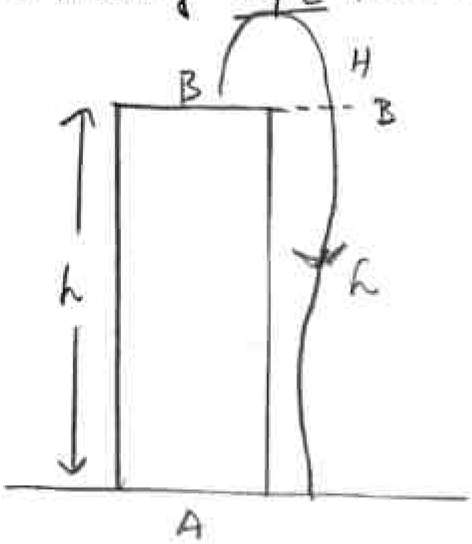
$$= \frac{2u \pm 2\sqrt{u^2 - 2gh}}{2g}$$

$$t = \frac{\cancel{2}(u \pm \sqrt{u^2 - 2gh})}{\cancel{2}g}$$

$$\boxed{t = \frac{u \pm \sqrt{u^2 - 2gh}}{g}}$$

A helicoptar is assending vertically with the speed of $8 m/s$. At a height of 120m above the earth. ~~A~~ A ~~packeg~~ package is droped from a window ~~a~~ how much time does it takes for the ~~pa~~ package to reach the ground.

Note:- motion of the ~~do~~ body throune vertically up from top of the tower.



$H = -120m$

$AB = h$

$BC = H$

$CA = (h+H)$

$CD = h-(h+H)$

$= \cancel{h} - \cancel{h} - H$

$CD = -H$

$S = ut + \frac{1}{2}at^2$

$-4 = ut - \frac{1}{2}gt^2$

$$\boxed{H = -ut + \frac{1}{2}gt^2}$$

$h = ut - \frac{1}{2}gt^2$

$-120 = 8 \times t - \frac{1}{\cancel{2}} \times \cancel{9.8}^{4.9} \times t^2$

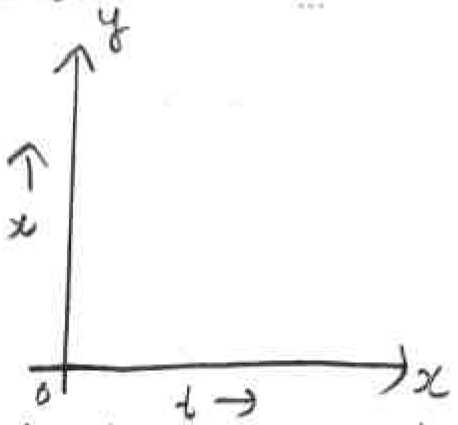
$-120 = 8t - 4.9t^2$

$4.9t^2 - 8t - 120 = 0$

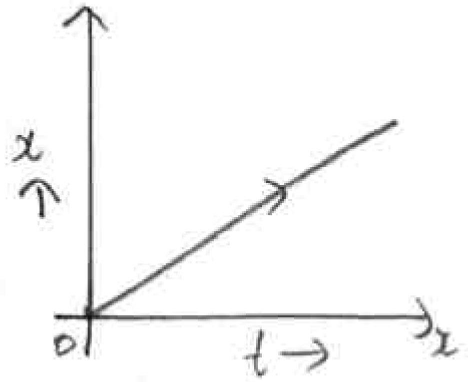
$t = 5.83 sec$

* ~~Graf~~ Graphical representation of motion of body

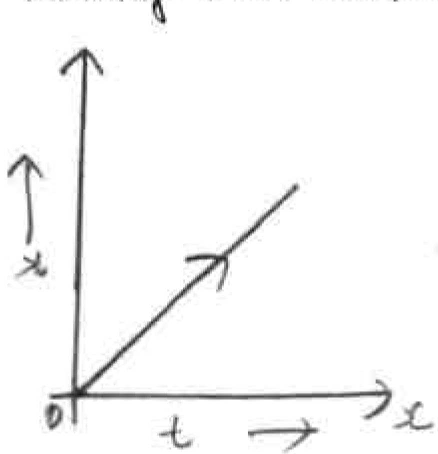
→ distance - time.



1) If the object is in rest

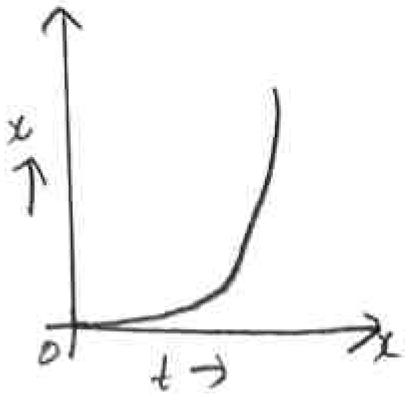


2) object is moving with uniform speed.



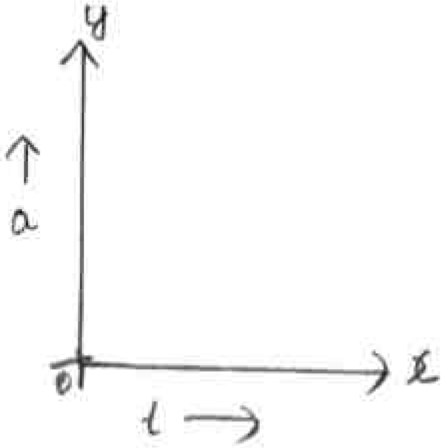
3) If the object moving in uniform acc

4)

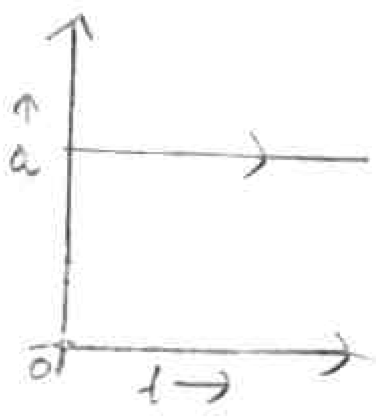


If Non uniform acc and speed.

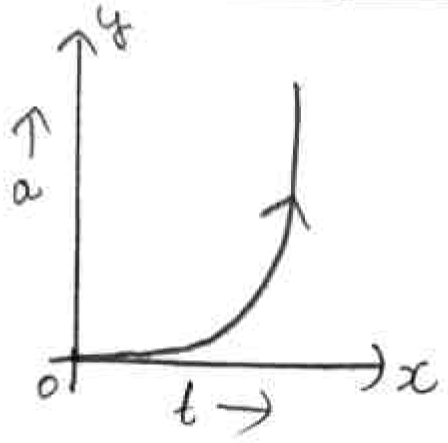
→ Acc vs time graph



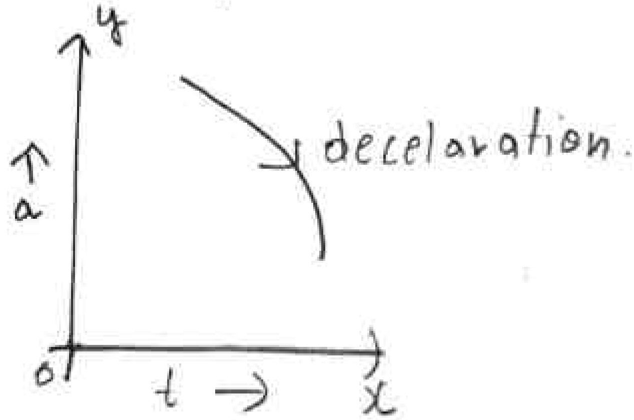
1) If the acc is zero.



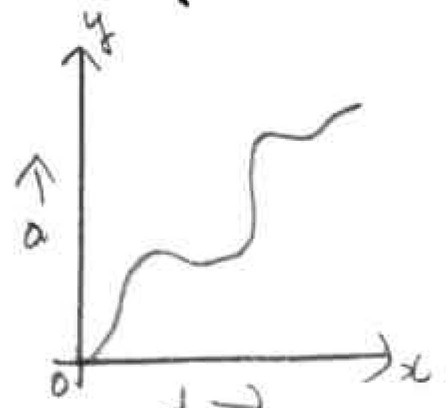
2) If the acc is constant.



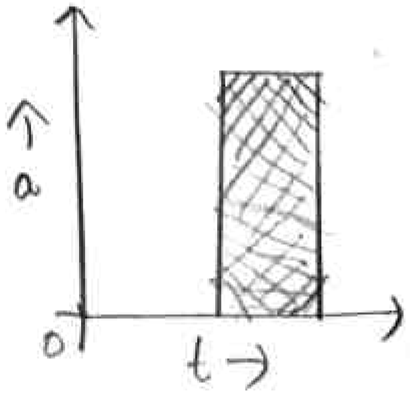
3) Increasing on acc



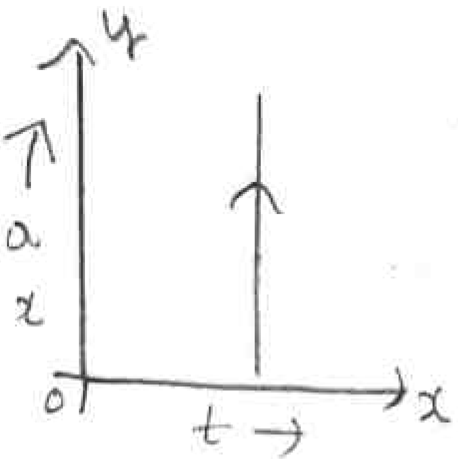
4) decreasing of acc



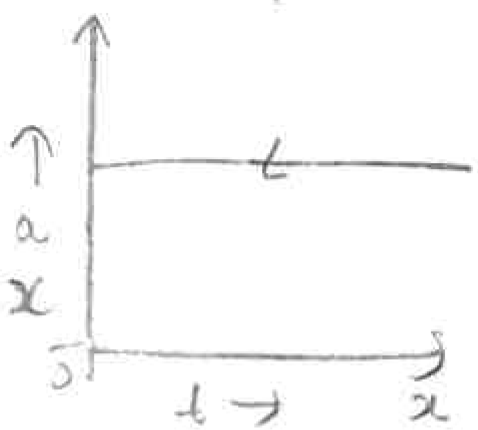
5) If the acc is zero at Initial, non uniform.



6) The chonge in velosity of the particals
= area enclosed b/w the time acc
curve and the time axis



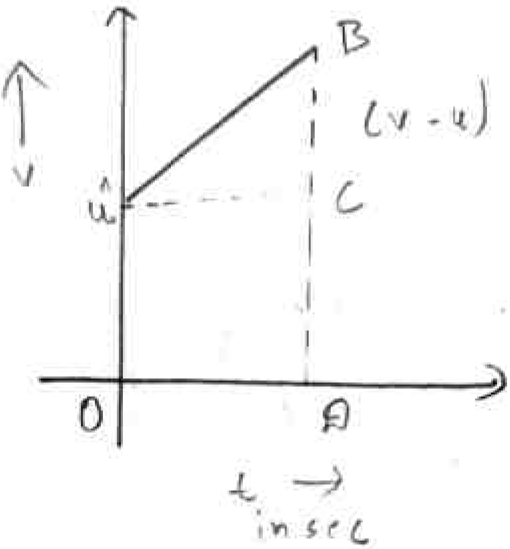
7) Such motions are not possible



8) Such motions are not posible

5marks → IMP

Derivation of eq of uniformly accelerated motion (Grapical method using VT graph)



Let us consider an object moves along a stright line with uniform acc 'A'. Its ~~ineti~~ inetial velocity u & v is its final velocity after time interval t. The distance traveled by the object during this time interval is 's'. velocity time graph of this motion is a stright line AB. Here OD = t = AC

AB = a

BC = V - u.

1) slop of velocity time graph gives acc of an object

v - t

$$acc = AB = \frac{BC}{AC} = \frac{V-u}{t}$$

$$a = \frac{V-u}{t}$$

$$at = V - u \qquad \boxed{V = u + at} \rightarrow ①$$

2) $S = ut + \frac{1}{2} at^2$

The area under the velocity time graph for a given time interval represents the distance covered by the object in the time interval

distance traveled by the object ~~in~~ in time t is OABD (trapezium).

2) displacement equation.

from OABD

$S =$ area of OABD (trapezium)

$=$ area of Δ ABL + Area of $\square$ ACDO

$= \frac{1}{2} BC \times AC + AC \times CD$

$S = \frac{1}{2} (v - u) t + t \times u$

From eq ① $v = u + at$

$\boxed{v - u = at}$

Replace in above

$S = \frac{1}{2} \times at \times t + t \times u.$

$\boxed{S = ut + \frac{1}{2} at^2} \longrightarrow$ ②

3) $2as = v^2 - u^2$ (Area of trapizium displacement, s)

displacement, s

$s = OABD$ Trapizium {area}

$s = \frac{1}{2}(OA + DB) \times DO$

$s = \frac{1}{2}(u+v) \times t$

{ $\because$ area of trapizium $= \frac{1}{2} \times$ (sum of two side) $\times$ distance b/w 2 sides)

$\therefore$ from 1^{st} eqn

$v = u + at$

$v - u = at$

$t = \frac{v-u}{a}$

substitute in the above eq.

$s = \frac{1}{2}(u+v) \times \frac{(v-u)}{a}$

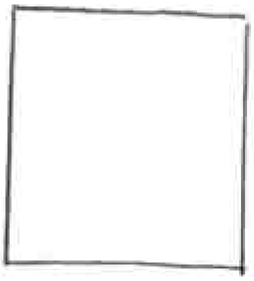
$s = \frac{1}{2a}(u+v)(v-u)$ { $(a+b)(a-b) = a^2 - b^2$

$2as = (v+u)(v-u)$

$\boxed{2as = v^2 - u^2} \rightarrow 3$

→ Example - 4.1

Find the distance covered by the particle in the following figures (i) and (ii) while a particle travels from A to B as shown by the arrows.



i) Actual length $= AC + CD + DB$

$$= b + l + \frac{\sqrt{b^2+l^2}}{2}$$

$$\triangle \quad \frac{AD}{2} = \frac{\sqrt{b^2+l^2}}{2}$$

ii) actual length $= AC + CD + DB$

$$= a + \frac{\pi a}{2} + a$$

$$= 2a + \frac{\pi a}{2}$$

$$= \frac{4a + \pi a}{2}$$

$$\text{distance} = \frac{a(4+\pi)}{2}$$

$$= 2a + \frac{\pi a}{2}$$

Example 4.3

The distance travelled by a particle in time t is given by $s = (2.5)t^2$. Find (a) the avg speed of the particle during the time 0 to 5.0 s, and (b) the instantaneous speed at $t = 5.0$ s. Here s is in metres.

→ Example 4.4

A bus b/w vijayawada and Hyderabad passed the 100 km, 160-km and 220-km points at 10.30 a.m. 11.30 a.m and 1.30 pm Find the avg speed of the bus during each of the following intervals.

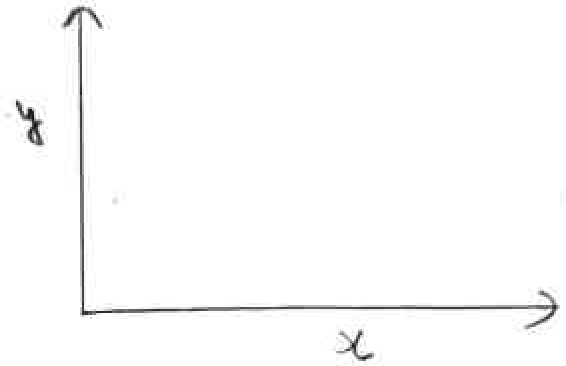
a) 10.30 a.m to 11.30 am.

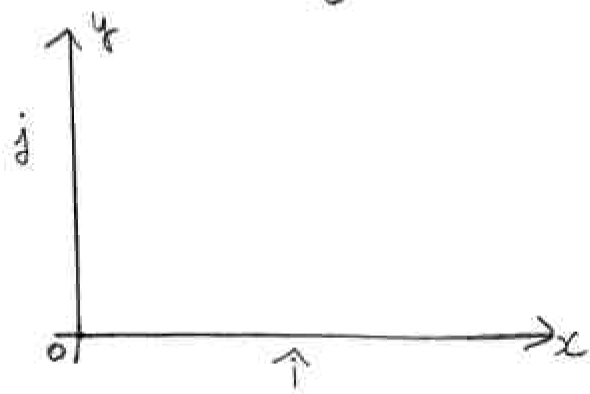
b) 11.30 am to 1.30 pm and

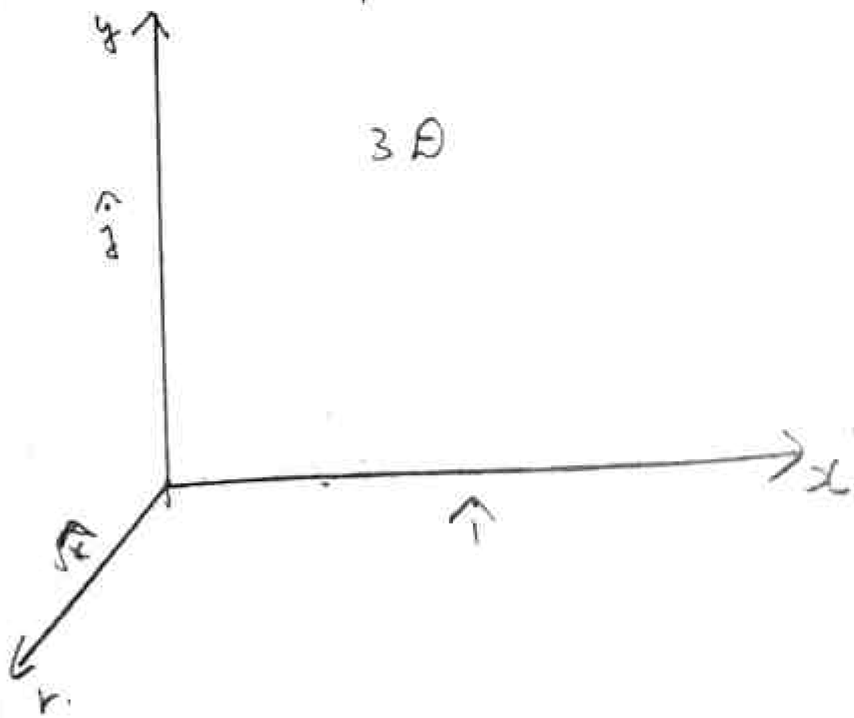
c) 10.30 am to 1.30 p.m.

Motion in a Plane.

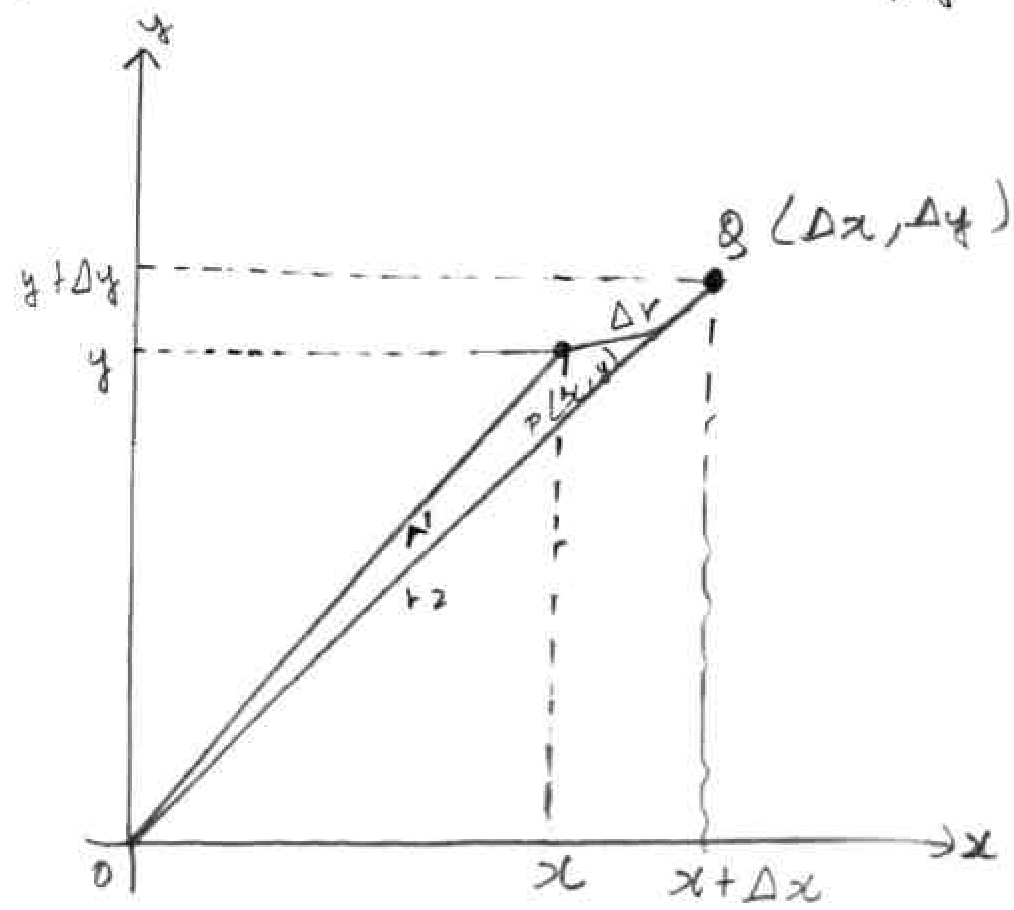
→ Motion ina ~~plain~~. plane.







→ If a partical is free to move in a plane its position can be located with two co-ordinates We choose the plane of motion as the xy plane. The position of the partical at a time t is completly denoted by its ~~cordenated~~ co-ordinate x, y



Interms of velocity $\boxed{\vec{v} = v_x \hat{i} + v_y \hat{j}}$

Position vector

The position vector of an object in x,y plane is given by $r = \cancel{xy}\ x\hat{i} + y\hat{j}$ ~~and~~

displacement vector

The displacement vector from position r_1 to r_2 $\boxed{\Delta r = r_2 - r_1}$

$$= (x - \Delta x) - (y - \Delta y)$$

$$= \Delta x\hat{i} - \Delta y\hat{j}$$

$$r = x\hat{i} + y\hat{j}$$

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$v_x = \frac{dx}{dt}\hat{i} \qquad v_y = \frac{dy}{dt}\hat{j}$$

diff w.r.t

$$\frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

3D displacement vector.

$r = x\hat{i} + y\hat{j} + z\hat{k}$

diff w.r.t

$\frac{dr}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$.

In velocity $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

diff w.r.t

$\frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x)\hat{i} + \frac{d}{dt} v_y\hat{j} + \frac{d}{dt} v_z\hat{k}$

$\boxed{\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}$

→ The kinamatic eq based on x & y components

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 - u^2 = 2as$

x - components.

$v_x = u_x + a_x t$

$s = u_x t + \frac{1}{2} a_x t^2$

$v_x^2 - u_x^2 = 2a_x s$

y - component.

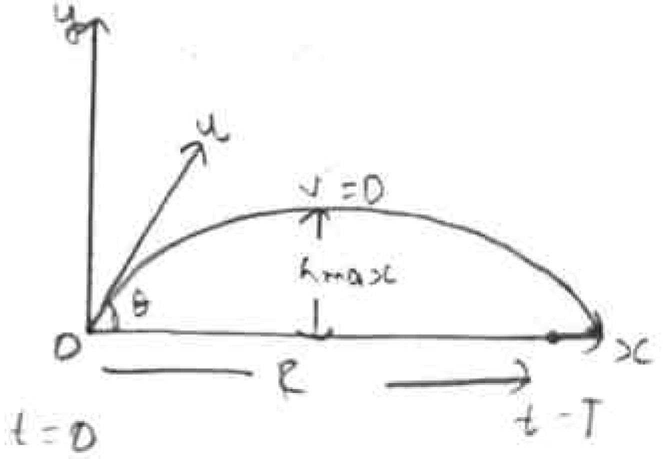
$v_y = u_y + a_y t$

$s = u_y t + \frac{1}{2} a_y t^2$

$v_y - u_y = 2a_y s$.

→ Projectile motion.

Projectile in any body projected into the body into air at an angle other than 90^0 write Horizontal near the path followed by the ~~object~~ projectile. is called trajectory (Path.



$a = -g$

consider the motion of an object Projected from the ~~origin~~ orgin O of the co-ordinate system write the initial velocity inclined at an angle θ ~~uniform~~ with the horizontal or x axis is shown in the figure

The motion of the object P can be resolving by its velocity 'u' in the horizontal and the vertical components.

→ Horizontal component along x-axis = $u\cos\theta$
vertical component along y-axis = $u\sin\theta$
acc. due to gravity is '$-g$'

let 'p' be the position of the object after a time 't' then distance travelled in the horizontal direction in time t,

along x-axis the Horizontal component

$$v = \frac{x}{t}$$

$$u\cos\theta = \frac{x}{t}$$

$$\boxed{x = t \cdot u\cos\theta}$$

$$\boxed{t = \frac{x}{u\cos\theta}}$$

trajectory made by the object the distance travelled by the object
Path travelled by the object.

$$s = ut + \frac{1}{2}at^2$$

$s = y$
$a = -g$.

$$y = u\sin\theta \times \frac{x}{\cancel{u}\cos\theta} + \frac{1}{2}(-g)\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = \tan\theta\, x - \frac{g}{2}\,\frac{x^2}{u^2\cos^2\theta}$$

$$y = \tan\theta\, x - \frac{g}{2u^2\cos^2\theta}x^2$$

$y = Ax + Bx^2$

$\boxed{y = Px + qx^2}$

Path followed by the object is Parabola.

a) time taken to reach maximum height or time of ascent

~~$t_a = ?$~~

→ Let the time be denoted as 't_a' (time of ascent) at the maximum height the vertical component of velocity is zero. $v = 0$

$t_a = ?$ $\quad v = u + at$

$0 = u + (-g)\,t_a$

$0 = u - g t_a$

$g t_a = u$

$\therefore t_a = \frac{u}{g}$

If it is verticle

$$\boxed{t_a = \frac{u \sin\theta}{g}}$$

b) Time of flight.

The total time spent by the object of Project in ~~are~~ air during the motion is called time of flight.

$$S = ut + \frac{1}{2}at^2$$

$S = 0$

$a = -g$

$t = T_f$

$$0 = u\sin\theta T_f + \frac{1}{2}(-g)T_f^2$$

$$\frac{1}{2}gT_f^{\cancel{2}} = u\sin\theta \cancel{T_f}$$

$$\boxed{T_f = \frac{2u\sin\theta}{g}}$$

c) Maximum height

As the projectile moves up, the vertical component of its velocity decreases. At maximum height the vertical componet of velocity becomes zero.

$S = H_{max}$

$u = u\sin\theta$

$v = 0$

$a = -g$

From eq ③

$$v^2 - u^2 = 2as$$

$$0^2 - u^2\sin^2\theta = 2(-g)H_{max}$$

$$\cancel{-}u^2\sin^2\theta = \cancel{-}2g\,H_{max}$$

$$\boxed{H_{max} = \frac{u^2\sin^2\theta}{2g}}$$

d) Horizontal range

The Horizontal distance traveled by the projectile wile it ~~tooks~~ touchs the same level of the point of projection is called Horizontal range.

$$R = \text{Horizonted component} \times T_f$$

$$= u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$= \frac{u^2}{g} 2\sin\theta \times \cos\theta$$

$$\boxed{R = \frac{u^2}{g}\sin 2\theta}$$

IMP | Motion in a stright line.

1) Write any ~~two~~ three ~~b/w~~ differencens b/w distance & displacement

2) velocity & speed

3) define

a) avg speed , b) instantaneous velocity

c) accelaration d) avg accelaration

4) VT Graph.

a) when the object is at rest

b) " " " uniform motion & constant

c) " " " increasing of acc & decreasing of acc.

5m 5) derive the eq of motions by ~~integral~~ calculus methord

6) derive the eq of motions using VT gra

7) ~~define~~

Ch-2 Motion in a plain

1) Define projectile motion
2) Time of flight
3) Maximum height
4) Horizontal range.

With expresions

7) prove that an object having the projectile in tragectrimotion.

1) A cricet ball is thrown at a speed 28 m/s. in a direction 30° above the horizontal. calculate. ~~first (A)~~

A) Maximum height B) The time taken by the ball to return to the same level

C) The distance from the ~~thror the~~ thrower ~~the~~. to the where the ball returns to the same level.

Note:- If $\theta = 30^\circ \rightarrow$ above the Horizontal

$\theta = 30.$

If $\theta = 30^\circ \rightarrow$ above the vertical line.

$\theta = 90^\circ - \theta$

$= 90 - 30$

$\boxed{\theta = 60^\circ} \rightarrow$ Horizontal angle

$u = 28$ m/s

$$h_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{28^2 \times \sin^2 30^\circ}{2 \times 9.8}$$

$$= \frac{784 \times \left(\frac{1}{2}\right)^2}{19.6}$$

$$= \frac{784 \times \frac{1}{4}}{19.6} = \frac{196}{19.6} = 10m$$

$\cancel{R = \frac{u^2}{g}}$

$$T_f = \frac{2u\sin\theta}{g}$$

$$= \frac{2 \times 28 \times \sin 30}{9.8}$$

$$= \frac{\cancel{2} \times 28 \times \frac{1}{\cancel{2}}}{9.8}$$

$$= \frac{\cancel{2} \times 28 \times \frac{1}{\cancel{2}}}{9.8}$$

$$T_f = \frac{28}{9.8} = 2.85\ \text{sec}$$

$$\boxed{R = 69.2\ m}$$

$$R = \frac{u^2}{g}\sin 2\theta$$

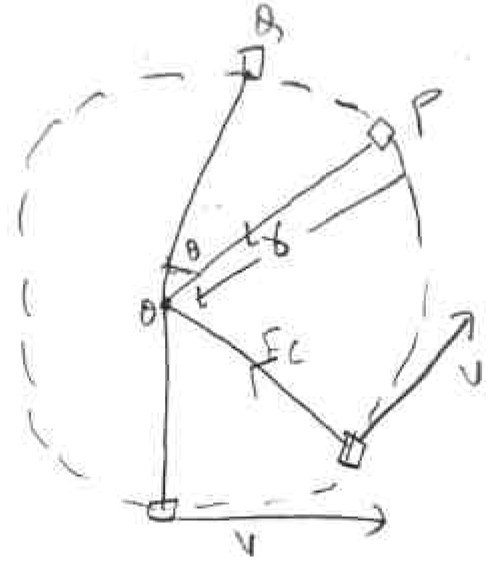
$$= \frac{(28)^2}{9.8} \times \sin 2 \times \sin$$

$$= \frac{\overset{392}{784}\ \sqrt{3}/2}{9.8}$$

$$= \frac{392 \times 1.73}{9.8}$$

* Uniform cirqular motion

when an object followes the circular path at constant speed. with vareable velocity of an object is known as uniform cirqular moti...



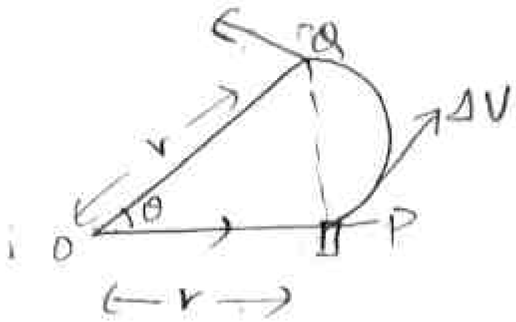
Let us consider an object moving in a circular path with uniform speed. and having change in direction in every position. with respect to time and having a radius 'r'

here 'P' is the first position of an object & 'Q' is the another position of an object the position of an object ~~wil~~ is changing then the angle of the every insentonews position will change.

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t}} \rightarrow \text{angular velocity}$$

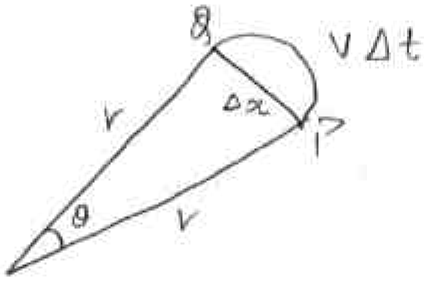
To find centeralfital acc

from the uniform cirqutar motion the speed of an object is constant and the object changes the direction of the object in circular path.



From velocity vector

$\rightarrow \frac{\Delta V}{V} \rightarrow$ ①



From radial vector

$\frac{V\Delta t}{r} \rightarrow$ ②

Equating eq ① = ②

$$\frac{\Delta V}{V} = \frac{V\Delta t}{r}$$

$$\frac{\Delta V}{\Delta t} = \frac{V \times V}{r}$$

$$\frac{\Delta V}{\Delta t} = \frac{V^2}{r}$$

centripetal acc

$$\boxed{a_c = \frac{v^2}{r}}$$ → This is the centripetal acc

~~From Newton~~

To find centripetal force.
From Newton 2nd law

$$F = ma$$

$$\boxed{F_c = \frac{mv^2}{r}}$$

→ Relationship b/w angular velocity and linear velocity.

~~$v = \frac{s}{t}$~~

The linear velocity of an object is defined as. $\frac{s}{t}$

~~$v = \Delta s$~~

$v = \frac{s}{t}$

The angular velocity of a object is defined as $\frac{\Delta\theta}{\Delta t}$

$v = \frac{\Delta s}{\Delta t}$ → linear velocity

from uniform cirqular motion the position, the arc length will be changes with respective time and radias is constant.

$$\theta = \frac{\text{arc length}}{\text{radius}}.$$

$$\Delta\theta = \frac{\Delta s}{r} \rightarrow (2)$$

$$r\Delta\theta = \Delta s$$

$$v = \frac{r\Delta\theta}{\Delta t}$$

$$\boxed{v = \omega r}$$

$$\omega = \frac{v}{r}$$

$$a_c = \frac{v^2}{r}$$

$$= \frac{\omega^2 r^2}{r}$$

$$\boxed{a_c = \omega r^2}$$

angular acc

$$F = ma$$

$$\boxed{F = mr\omega^2}$$

angular force.

→ Angle of projection for the maximum range of projectile

$$R = \frac{u^2 \sin 2\theta}{3}$$

The range 'R' ~~from~~ for a given velocity 'u' is maximum if $\sin 2\theta = 1$ are $2\theta = \frac{\pi}{2}$

When $\theta = \frac{\pi}{4} = 45^\circ$

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{u^2 \sin 2\left(\frac{\pi}{4}\right)}{g}$$

$$= \frac{u^2 \sin\left(\frac{\pi}{2}\right)}{g}$$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

Relation b/w Range & maximum height

Range of an object $R = \frac{u^2 \sin 2\theta}{g}$

&

maximum height $= \frac{u^2 \sin^2\theta}{2g}$

(1).

$$\frac{H}{R} = \frac{\frac{u^2\sin^2\theta}{2g}}{\frac{u^2\sin 2\theta}{g}} = \frac{\frac{u^2\sin^2\theta}{2g}}{\frac{u^2 2\sin\theta\cos\theta}{g}}$$

$$= \frac{u^2\sin^2\theta}{2\cancel{g}} \times \frac{\cancel{g}}{\cancel{u^2}\,2\,\cancel{\sin\theta}\times\cos\theta}$$

$$= \frac{1}{4} = \frac{\sin\theta}{\cos\theta}$$

$$4H = R\tan\theta$$

$$\frac{H}{R} = \frac{1}{4}\tan\theta$$

$$\boxed{R = \frac{4H}{\tan\theta}}$$

$$\boxed{H = \frac{R}{4}\tan\theta}$$

$$\boxed{\tan\theta = \frac{4H}{R}}$$

→ There are two angles of projection for same range

$$\theta = 90^\circ - \theta$$

$$R = \frac{u^2\sin 2\theta}{g}$$

$$= \frac{u^2\sin 2(90^\circ-\theta)}{g} = \frac{u^2\sin 2\theta}{g} = R \to ①$$

$$= \frac{u^2\sin(180^\circ-2\theta)}{g} = \frac{u^2\sin 2\theta}{g} = R \to ①$$

eq ① = ②

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

→ Relation b/w times of flights and range in case of projectile having same range

If θ

$$T_1 = \frac{2u\sin\theta}{g} \rightarrow (1)$$

$$\theta = (90-\theta)$$

$$T_2 = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g} \rightarrow (2)$$

$$T_1T_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g}$$

$$= \frac{2u^2[2\sin\theta\cos\theta]}{g\quad[g]}$$

$$= \frac{2u^2}{g}\left(\frac{\sin 2\theta}{g}\right)$$

$$R = \frac{u^2}{g}\sin 2\theta$$

$$T_1T_2 = \frac{2}{g}R \rightarrow (3)$$

$$\frac{T_1}{T_2} = \frac{2u\sin\theta/g}{2u\cos\theta/g}$$

$$\frac{T_1}{T_2} = \frac{\cancel{2u}\sin\theta}{\cancel{g}} \times \frac{g}{\cancel{2u}\cos\theta}$$

$$\boxed{\frac{T_1}{T_2} = \tan\theta} \qquad \theta = \tan\left(\frac{T_1}{T_2}\right)$$

* ~~Laws of motion~~ Ch-6 Laws of motion

→ Linear movementum :- The ~~moveme~~ movementum a body is defined as the product of m of the body & its velocity

$P = M \times V$

It is a vector quantity.

SI unit = kg m/s

dimentional formula = $[M^1 L^1 T^{-2}]$

→ Newtons laws of motion

Newton gave three laws of motion of bodys.

First law :- Every body continues to be in its state of rest ~~are~~ or uniform motion along a stright line, unless it acted upon by some external force) to change the state

Eg :- i) A book lies on a table,

ii) A bus is moving in uniform speed

Secound law :- The rate of change of linear movementum of a body is directly propostional to the external force applied on the body, and this change takes place in the direction of force applied

Eg : i) Pushing a cart

ii)

third law :- For every action there is an equal & opposite reaction that is the force of action & reaction are always equal & opp & acting on different ~~bodges~~ bodies and having same nature.

Eg :- i) Throughing a ball to wall with the force & it will return with new force.

note :- convertion
m/s → x × $\frac{18}{5}$ kmph
kmph → x × $\frac{5}{18}$ m/s

→ Inertia of a body

It is of three types.

i) Inertia of Rest

ii) Inertia of motion

iii) Inertia of direction

i) Inertia of rest :- It is the inability of the body by its change place by its self h

Eg :- When a horse startes suddenly, the rider tends to ~~po~~ fall backwords on accaunt of inertia of Rest.

ii) Inertia of motion :- It is the inability of the body to change by itself it states of uniform motion.

Eg :- Suppose we are standing in a moving bus the driver stoped the bus suddenly we are thrown forword with the jurk

iii) Inertia of direction:- It is inability of to change by itself its direction of motion.

Eg:- An umbrella protects us from rain. it is based of properties of Inertia of direction the rain dropes falling vertically downwa... cannot change there direction ~~and we~~ of motion, wet us. with umbrella on

★ Explanation for newton's second.

According to newton's secound law of motion the rate of change of linear movementum of the body is directly proposal to the external force applied.]

conside a body of mass 'm' moving with velocity 'v' the linear movement of a body $p = m \times v$, f = external force
ΔP is small change in linear momentum of the body in a small time Δt
From Newton's second law

$$F_{ext} \propto \frac{dp}{dt} \qquad P = mV$$

$$F_{ext} \propto \frac{d}{dt} mV$$

$$F_{ext} \propto \frac{m\,dv}{dt}$$

$$a = \frac{dv}{dt}$$

$$\angle F_{ext} = \lim_{\Delta t \to 0} \frac{\Delta P}{\Delta t} = \frac{dP}{dt}$$

$F_{ext} \propto m\vec{a}$

$F_{ext} = km\vec{a}$

$\boxed{k = 1}$

$\boxed{F = m\vec{a}}$

SI unit $kgms^{-2}$

$DEF = [M^1L^1T^{-2}]$

Continuation

Rate of chang of linear momentum of the body ~~ΔP by Δ~~ $\frac{\Delta P}{\Delta t}$

~~acce~~ $\therefore$ the magnitude of force can be calculated by multipled mass of the body and acc produced in it.

Dimentional Formula of force $\therefore [M^1L^1T^{-2}]$

SI unit of force is

$F = ma$

$= mass \times acc$

$= kg \times ms^{-2}$

$1N = 1 kg\, ms^{-2}$

convert N into dyne

$n_1 u_1 = n_2 u_2$

$1N = kg\, ms^{-2}$

$= 1000 g \times 100 cm \times 1 s^{-2}$

$= 10^3 g \times 10^2 cm \times 1 s^{-2}$

$= 10^5 g\, cm s^{-2}$

$1N = 10^5$ dyne.

* Gravitational ~~force~~ unit

→ 1 kg force. It is that much force which produces an acc of $9.8 m/s^2$ in a body of mass 1 kg.

$W = mg$

$$1 kg\ wt = 1 kg \times 9.8 m s^{-2}$$
$$= 1 \times 9.8 kg\ m s^{-2}$$
$$= 9.8 kg\ m s^{-2}$$
$$\boxed{1 kg\ wt = 9.8 N}$$

2) $$1 g\ wt = 1 g \times 9.8 m s^{-2}$$
$$= 1 g \times 9.8 \times 10^2 cm \times s^{-2}$$
$$= 1 \times 9.8 \times 100 \times g\ cm\ s^{-2}$$
$$\boxed{1 g\ wt = 980 g\ cm^{-2} s}$$

$$1 g\ wt = 980 \text{ dyne.}$$

6.1 A body of mass 8kg . is moved by . a force $f = 3x\,N$, where x is the distance covered. Ineatial position is $x = 2m$ & final position $x = 10m$, If ineballly a body is at rest find the final speed

$F = ma$

$a = \frac{dv}{dt}$

$F = m\frac{dv}{dt}$

$x_i = 2m$

$x_f = 10m$

$F = 3xN$

$m = 8kg$.

$F = m\frac{dv}{dt} \times \frac{dx}{dx}$

$F = m \cdot \frac{dv}{dx} \cdot \frac{dx}{dt}$

$F = m\frac{dv}{dx} \cdot V$

$F = dx = m.v.dv \rightarrow (1)$

$$\int_{x_1=2}^{x=10m} 3x\,dx = \int_0^{v} 8v\,dv$$

$$3\int_2^{10} x\,dx = 8\int_0^{v} v.dv$$

$$3\left[\frac{x^2}{2}\right]_2^{10} = 8\times\left[\frac{v^2}{2}\right]^{v}$$

$$3\left[\frac{10^2}{2} - \frac{2^2}{2}\right] = 8\times\left[\frac{v^2}{2} - \frac{0^2}{2}\right]$$

$$\frac{3}{\cancel{2}}(100-4) = \frac{8}{\cancel{2}}[v^2]$$

$100 - 4 = 8v^2$

$96 = 8v^2$

$V^2 = \frac{96\times 3}{8}$

$v^2 = \frac{288}{8}$

$v^2 = 36$

$v = \sqrt{36}$

$\boxed{v = 6\,m/s}$

6.2 If a bullet of mass 5 g. ~~g~~ moving with the velocity of 100 m/s penitrat a wooden block of upto 6 cm. find the avg force ~~in~~ in impose.

$v^2 - u^2 = 2as$

$0^2 - (100)^2 = 2 \times a \times 6 \times 10^{-2}$

$-10000 = 12a \times 10^{-2}$

$-\frac{10^{+4}}{10^{-2}} = 12a$

$-10^{+6} = 12a$

$-10^{+6} = 12a$

$-\frac{10^{+6}}{12} = a$

$a = \frac{-1}{12} \times 10^6$

$m = 5g$

$F = ma$

$= 5 \times 10^{-3} \times$

$(-0.08\ldots)$

$F = 0.416 \times 10^{+3}$

$\boxed{F = 416 N}$

6.3

→ A Gravel is dropped on a conveyer belt at the rate of 0.5 kg/sec. Find the extra force required in Newton to keep the belt moving at 2 m/sec

$$\frac{dm}{dt} = 0.5 \text{ kg/s}$$

$$v = 2 \text{ m/s}$$

$$F = ma$$

$$= m\frac{dv}{dt}$$

$$F = \frac{dm}{dt} \cdot v$$

$$F = 0.5 \times 2$$

$$\boxed{F = 1N}$$

6.4

→ A body of mass 5 kg starts from the origin with an initial velocity of $\bar{u} = (30\hat{i} + 40\hat{j})$ m/s A constant force of $F = (-\hat{i} - 5\hat{j})$ N acts on the body find the time in which the y-component of the velocity becomes zero.

$$u = (30\hat{i} + 40\hat{j}) \text{ m/s}$$

$$F = (-\hat{i} - 5\hat{j}) N$$

y - co-ordinates

$$u = (u_x\hat{i} + u_y\hat{j})$$

$$F = (F_x\hat{i} + F_y\hat{j})$$

$u_y = 40 m/s$

$F_y = -5 N$

$m = 5 kg$

$v = 0$

$F_y = ma_y$

$-5\hat{i} = 5a_y$

$a_y = \frac{\cancel{-5}}{\cancel{5}}$

$\boxed{a_y = -1 m/s^2}$

$v_y = u_y + a_y t$

$0 = 40 + (-1) \times t$

$0 = 40 - t$

$\boxed{t = 40 s}$

x - co - ordinate

$u_x = 30\ m/s$

$m = 5kg$

$F_x = -1N$

$F_x = ma_x$

$-1 = 5 \times a_x$

$a_x = \frac{-1}{5}$

$a_x = 0.2\ m/s.$

$v_x = u_x + a_x t$

$0 = 30 + (-0.2)t$

$0 = 30 - 0.2t$

$0.2t = 30$

$t = \frac{30}{0.2}$

$\boxed{t = 150 sec}$

→ A body of mass 5 kg is moved by a force $F = 2x$ N where x is the distance covered. Initial position is $X = 2$ m and final position is $x = 10$ m. If initialy the body is at rest, find the final speed.

$F = ma$

$$F = m\frac{dv}{dt} \times \frac{dx}{dx}$$

$$F = m\frac{dv}{dx} v$$

$$F.dx = m.v\,dv \rightarrow (1)$$

$F = 2x$ N

$m = 5$ kg

velocity
$0 \rightarrow v$
distance
1 m $\rightarrow 8$ m.

$$\int^{2} F.dx = \int_{0}^{v} m.v.dv$$

$$\int^{8} 2x.dx = 5\int_{d}^{v} v.dv.$$

$$2\left[\frac{x^2}{2}\right]_1^8 = 5\left[\frac{v^2}{2}\right]_0^v$$

$$2\left[\frac{8^2}{2} - \frac{1^2}{2}\right] = 5\left[\frac{v^2}{2} - \frac{0^2}{2}\right]$$

$$\cancel{x}\left[\frac{64}{\cancel{x}} - \frac{1}{\cancel{x}}\right] = 5\left[\frac{v^2}{2}\right]$$

$$64 - 1 = 5\left[\frac{v^2}{2}\right]$$

$$63 = \frac{5v^2}{2}$$

$$63 \times 2 = 5v^2$$

$$126 = 5v^2$$

$$v^2 = \frac{126}{5}$$

$$v^2 = 25.2$$

$$v = \sqrt{25.2}$$

$$\boxed{v = 5.01\ m/s}$$

6.b. A satellite in force free space sweeps stationary interplanetary dust at a rate $\frac{dm}{dt} = \alpha V$ where m is mass, V is the velocity of the ~~stat~~ satellite and α is a constant. What is the deceleration of the satellite.

$$\frac{dm}{dt} = \alpha V$$

$$F = ma$$

$$= m\frac{dv}{dt}$$

$$F = \frac{dm}{dt} V$$

$$F = \alpha V . V$$

$$F = \alpha V^2 \rightarrow (1)$$

From 2nd law

$$F = ma$$

$$ma = \alpha v^2$$

$$\boxed{a = \frac{\alpha v^2}{m}}$$

* Springs.

$F \propto x$

$F = -kx$

$k \rightarrow$ spring constant

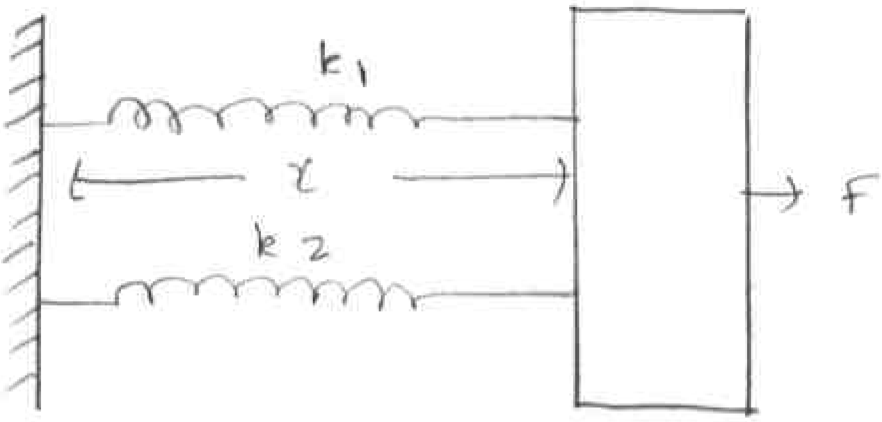
$F \rightarrow$ spring force

$x \rightarrow$ displacement by spring

A spring is fixed in one end. The other end of the spring is pulled by applying a force as ~~applyed~~ applied force increases, the elongation of the spring increases. a restoring force develops in the spring opposite to the elongation At any instant restoring force is directly proportional to the elongation.

{ }

* springs connected in parallel



If external force ~~acts~~ e acts ~~of a~~ on a combination of springs such that they have same deformation. that they are connected in parallel. If k equivalent is ~~of~~ the ~~combine spring~~ equivalent force constant of the combined spring then.

Force on each spring.

$F_1 = -k_1 x$ → Force spring 1

$F_2 = -k_2 x$ → Force spring 2

$F_{eq} = -k_{eq} x$

$F_p = F_1 + F_2$

$-k_{eq} x = -k_1 x + (-k_2 x)$

$-k_{eq} x = -k_1 x - k_2 x$

$\cancel{-}k_{eq} \cancel{x} = -\cancel{x}(k_1 + k_2)$

$$\boxed{k_{eq} = k_1 + k_2}$$

→ Springs in series:-

If same force is transmitted in all springs ~~in all springs~~ then we say that springs are connected in series.

Force on each spring.

Spring-1 $F = -k_1 x_1$, $x_1 = \frac{-F}{k_1}$

Spring-2 $F = -k_2 x_2$, $x_2 = \frac{-F}{k_2}$

$F_{eq} = -k_{eq} x$ $\qquad x = \frac{-F}{k_{eq}}$.

$x = x_1 + x_2$

$\frac{-F}{k_{eq}} = -\frac{F}{k_1} + \left(-\frac{F}{k_2}\right)$

$\frac{-\cancel{F}}{k_{eq}} = -\cancel{F}\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

$$\boxed{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}}$$

* Frome of Reference.

→ A frame in which an observer is situated and makes his observations is known as the frame of reference.

→ Frame of reference is of two types
i) Inertial frome of reference.
ii) Non - Inertiol frome of reference.

i) Inertiol frame of reference.

→ A frome of reference which is at rest or moving with the uniform velocity along a straight line is called an inertial frame of reference.

$A = 0, F = 0$

ii) Non-inertiol frame of reference.

→ Accelerated frame of reference is called Non-inertiol frome of reference.

$a \neq 0, F = 0$

* Requirement of Parade Force

* IMPULSE

large force.

$I = F \times t$

bomb blast

Ns

catching a ball

$M^1 L^1 T^{-1}$

$kg\,m\,s^{-1}$

From Newton's 2^{nd} law

$$F = \frac{dp}{dt}$$

$P_1 \to P_2$

$0 \to t$

$$F \cdot dt = dp$$

$$\int^{t} F \cdot dt = \int_{P_1}^{P_2} dp$$

$$F \cdot t = P_2 - P_1$$

$$\boxed{I = P_2 - P_1}$$

$I = P_2 - P_1$

suppouse which acts on a body for short time are called impulsive force

Eg':- i) In hitting a ball with bat.

ii) Bomb blast.

iii) In firing a gun.

iv) Accident of cars.

Impulse of a fore is measure of total effect of a force. It is given by the product of a force and the time for which the force acts ~~on~~ on a body. ~~Im~~

Impulse = force × time.

According to Newton's 2nd one

A bullet of mass 0.04 kg. moving with the speed of 90m/s enters a ~~havo~~ heavy ~~wood~~ wooden block & stopped after a distance of 60 cm. what is the avg resistive force exerted by the ~~bo~~ ~~bo~~ block ~~ano~~ on the bullet.

$m = 0.04\, kg$

$u = 90\, m/s.$

$S = 60\, cm$

$F = ?$

$v^2 - u^2 = 2as$

$0^2 - 8100 = 2 \times a (60/10^{-2})$

$$\frac{-8100 = 0}{\cancel{120} \times 10^{-2}}$$

$a = -67.50 \times 10^{+2}$

$\boxed{a = -6750\, ms^{-2}}$

$F = 0.04 \times -6750$

$\boxed{F = -6749.96\, N}$

Explanation of Newtons third law of motion

→ According to Newtons third law to every action there is a equal and opposite reaction the force exerted by the secound body on the first ~~F~~ F_{ab} is the force exerted on the body A by body B [Action]. And F_{BA} exerted on the body. ~~B~~ B by body A

$$\boxed{\vec{F}_{AB} = -\vec{F}_{BA}}$$

$$\vec{F}_{AB} + \vec{F}_{BA} = 0$$

$$\frac{\Delta P_1}{\cancel{\Delta t}} + \frac{\Delta P_2}{\cancel{\Delta t}} = 0$$

$$\Delta P_1 + \Delta P_2 = 0$$

$$\Delta P_1 = -\Delta P_2$$

$$P = mV$$

$$mV_1 = -mV_2$$

$$\boxed{\frac{dP_1}{dt} = \frac{-dP_2}{dt}.}$$

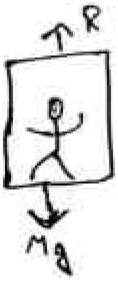
Elutions of Newton's third law

Eg:- i) A Book kept on a table

ii) Walking (iii) Swimming (iv) Firing from a Gun

v) Apparent weight of a ~~ball~~ man in lift

→ Case (i):- A man and ~~$~~ lift and rest



From 2nd law

$F = ma$

$a = 0$ $\boxed{R = mg}$

or

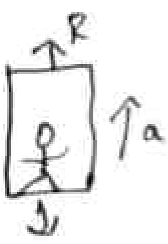
$R - mg = m(a)$ If $a = 0$

$R = mg = 0$

$\boxed{R = mg}$

→ Case (ii) man at Rest lift moves in upword direction

$\boxed{R > mg}$



From 2nd law

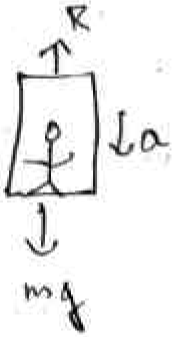
$F = ma.$

$R - mg = ma$

$R = ma + mg$

$R = m(a + g)$

→ Case (iii) a man at Rest, lift moves in downward direction.



$\boxed{Mg > R}$

2nd law

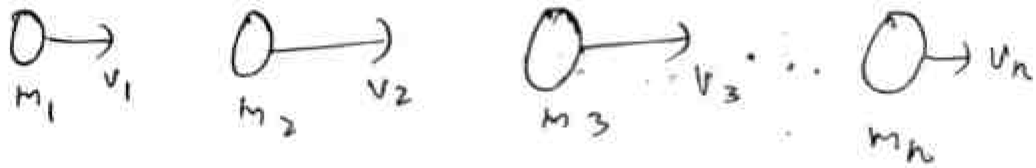
$F = ma$

$mg - R = ma$

$mg - ma = R$

$$\boxed{R = m(g-a)}$$

* The law of consermotion of linear movementum from Newton's secound law of motion.



The mass for the objects are

$m_1, m_2, m_3, \ldots \ldots m_n$

The velocity for the objects

$v_1, v_2, v_3, \ldots . V_n$

Total linear momentum of the objects.

$P_{net} = P_1 + P_2 + P_3 + \ldots + P_n \rightarrow (1)$

$mV = m_1 v_1 + m_2 v_2 + m_3 v_3 + \dots + m_n v_n$

$P = m_1 v_1 + m_2 v_2 + m_3 v_3 + \dots + m_n v_n$

$F = \frac{dp}{dt}$

From eq ①

$\frac{dp}{dt} = P_1 + P_2 + P_3 + \dots + P_n$

Isolated body (object is at rest)

$\boxed{\frac{dp}{dt} = 0}$ $\quad \frac{dP}{dt} = \text{constant}$

$\boxed{P = \text{Constant}}$

→ An object of mass 3 kg is at rest. Now a force $F = 6t^2 \hat{i} + 2t \hat{j}$ is applied on the object. Find the velocity of the object $t = 3s$.

$m = 3 \text{ kg}$

$F = 6t^2 \hat{i} + 2t \hat{j}$

$I = F \cdot t$

$\int_0^I dI = \int_0^3 F \cdot dt$

$I = \int_0^3 (6t^2 + 2t) \cdot dt$

$I = \left[\frac{\cancel{6}^2 t^3}{\cancel{3}} + \frac{\cancel{2} t^2}{\cancel{2}}\right]^3$

$I = \{2t^3 + t^2\}_0^3$

$I = [2(3)^3 + 3^2]$

$\boxed{I = 54\hat{i} + 9\hat{j}}$ $\quad u = 0$

$mv - mu = I$

$= 3(v - 0)$

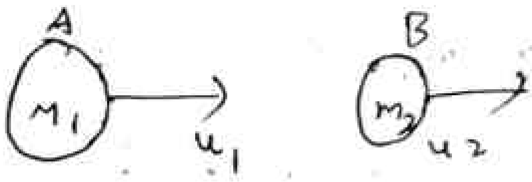
$= 3(v) = 54\hat{i} + 9\hat{j}$

$v = \cancel{54}^{18}\hat{i} + \cancel{9}^{3}\hat{j}$

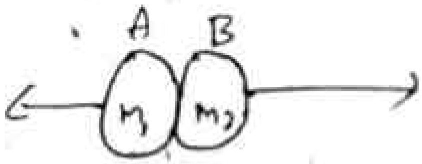
$\boxed{v = 18\hat{i} + 3\hat{j}}$ m/s

→ Derivation of the law of conservation of linear momentum for coliding bodys using Newton's third law

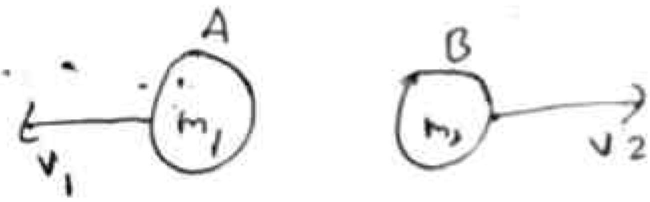
consider two bodys A, B of masses m_1 & m_2 moving in the same direction along a stright line with velocity u_1 & u_2 they colide for time Δt. After collution let their velocity v_1 & v_2



before collision



during collision



after collision

The force for first object A $= F_{AB}$
w.r.t 2nd object B

The force for second object B $= F_{BA}$.
w.r.t. 1st object A

$$\left\{\begin{aligned} \Delta t . F_{AB} &= m_1 v_1 - m_1 u_1 \\ \Delta t . F_{BA} &= m_2 v_2 - m_2 u_2 \end{aligned}\right.$$

From Newton's 3rd law
$F_{AB} = -F_{BA} \rightarrow (1)$

x'g by Δt on both sides

$$\Delta t\, F_{AB} = -\Delta t\, F_{BA}$$

$$m_1 v_1 - m_1 u_1 = -(m_2 v_2 - m_2 u_2)$$

$$m_1 v_1 - m_1 u_1 = -m_2 v_2 + m_2 u_2$$

$$\underbrace{m_1 v_1 + m_2 v_2} = \underbrace{m_1 u_1 + m_2 u_2}$$

linear moment for after collision = linear momentum for befor collision

$$\Delta t . F = \Delta P$$

$$\boxed{\Delta t . F = mV}$$

$\therefore$ linear moment of colliding bodies are conserved

~~$\Delta t\, F_{AB} = m_1 v_1 - m_1 u_1$~~

~~$\Delta t\, F_{BA} = m_2 v_2 - m_2 u_2$~~.

→ Recoial of a Gun

before firing = after firing .

$MV = -mv$

$MV + mv = 0$

$MV = -mv$

$\boxed{V = \frac{-mv}{M}}$ → velocity of gun w.r.t bullet .

$MV = -mv$

$\boxed{v = \frac{-MV}{m}}$ → velocity of bullet w.r.t gun

* Connected ~~body~~ bodies

→ case (i)

three bodies of masses m_1, m_2, m_3 are connected in series. A force 'F' is applied. The tension in b/w masses m_1, m_2, m_3.

$$F = ma$$

$$F = (m_1 + m_2 + m_3)a$$

$$a = \frac{F}{m_1 + m_2 + m_3}.$$

$$T_1 = m_1 a$$

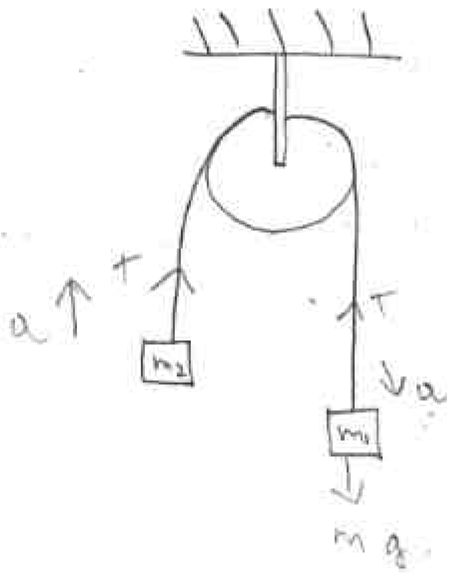
$$T_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)}$$

$$T_2 = (m_1 + m_2)a$$

$$T_2 = \frac{(m_1 + m_2)F}{(m_1 + m_2 + m_3)}$$

→ case - ii

Two bodies of mass m_1 & m_2 are connected by A light and inexpensibal string which passes over a ~~fire~~ friction less pully .



$m_1 > m_2$

$F = m_1 a$

$m_1 g - T = m_1 a \rightarrow (1)$

$F = m_2 a$

$T - m_2 g = m_2 a \rightarrow (2)$

From eq (1)

$T = m_1 g - m_1 a$

$T = m_1 (g - a) \rightarrow (3)$

From eq (2)

$T = m_2 a + m_2 g$

$T = m_2 (a + g) \rightarrow (4)$

$$m_1(g-a) = m_2(g+a)$$

$$m_1 g - m_1 a = m_2 g + m_2 a$$

$$m_1 g - m_2 g = m_1 a + m_2 a.$$

$$g(m_1 - m_2) = a(m_1 + m_2)$$

$$\boxed{a = \frac{g(m_1 - m_2)}{m_1 + m_2}.}$$

$$T = m_1(g-a)$$

$$= m_1\left(g - g\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\right)$$

$$= m_1 g\left(1 - \frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$T = m_1 g\left(\frac{m_1 + m_2 - (m_1 - m_2)}{m_1 + m_2}\right)$$

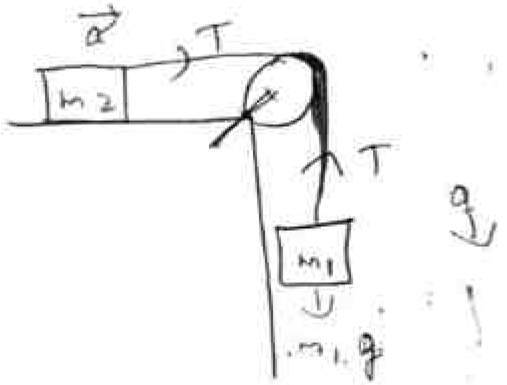
$$T = m_1 g\left(\frac{\cancel{m_1} + m_2 - \cancel{m_1} + m_2}{m_1 + m_2}\right)$$

$$T = m_1 g\left(\frac{2m_2}{m_1 + m_2}\right)$$

$$\boxed{T = \frac{2m_1 m_2 g}{m_1 + m_2}}$$

→ Case iii

Two bodies of masses m_1 & m_2 are connected by alight and extensibal string which ~~causes~~ passes over friction less pully. as shown in the fig. the surface on which m_2 ~~sil~~ slides is smooth.



$F = m_1 a$

$m_1 g - T = m_1 a \rightarrow$ ①

$T = m_2 a \rightarrow$ ②

Replace Tension in eq ①

$m_1 g - m_2 a = m_1 a$.

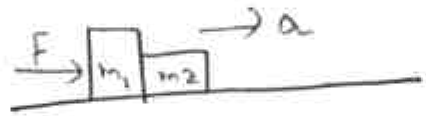
$m_1 g = (m_1 + m_2) a$.

$$\boxed{a = \frac{m_1 g}{m_1 + m_2}}$$

$T = m_2 a$

$$\boxed{T = \frac{m_2 m_1 g}{(m_1 + m_2)}}$$

Force of contact.



Two bodies of masses m_1 & m_2 are kept on a smooth ~~ofo~~ floor. A force 'F' is applied in the fig.

$$F = ma$$

$$F = (m_1 + m_2)a$$

$$a = \frac{F}{m_1 + m_2}$$

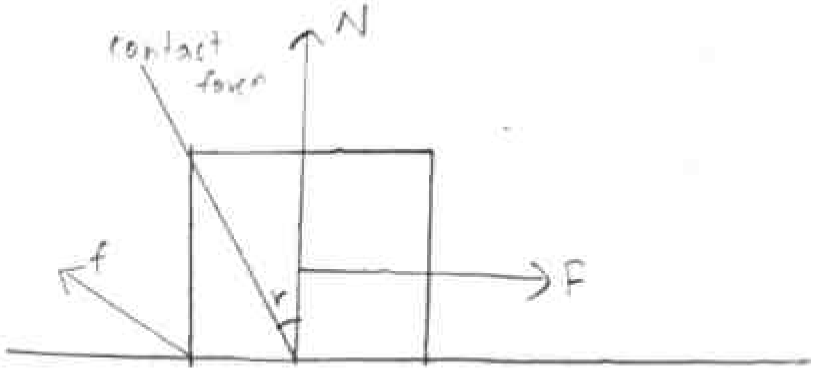
$$f = m_2 a$$

$$\boxed{f = \frac{m_2 F}{m_1 + m_2}}$$

✱ Friction

→ When two objects are kept one on the oth.. the actual area of contant will be much smaller then the total surface area of contact

A friction is a contact force it agsibates opposite to the normal force applied on an object.



f → friction

F → force

N → Normal Reaction

α → angle of friction

Friction may be defined as the opposing force which comes into plane tangentially b/w two Surfaces so as to destroy the relative motion b/w them.

* Advantages of Friction

i) safe walking on the floor is posibal becouse of friction b/w the ~~force~~ floor and feet.

ii) Friction helps the fingers to hold a drinking water tunbler or pen.

* Disadvantages of Friction.

i) The ~~wear~~ and tore of the machines increases due to friction reducing there life.

ii) Friction causes ~~unnec~~ uneccery heating of the parts of the machinary which may alter there opreating condetions.

* Types of Friction

i) Static Friction

ii) Kinetic Friction.

i) Static friction.

→ The resistance encountered by a body is static condetion wile tending to move under the action of an external force is called static friction.

static friction is denoted by $= f_s$

$$f_{max.s} = f_L$$

The maximum value of static friction is called limting friction

ii) kinetic friction.

→ The resistance encountered by a sliding body on a surface is known as kinetic friction. (or) dinamic friction.

It is denoted as = f_k.

* laws of Frictions

i) static friction.

→ static friction is directly proportional to normal reaction

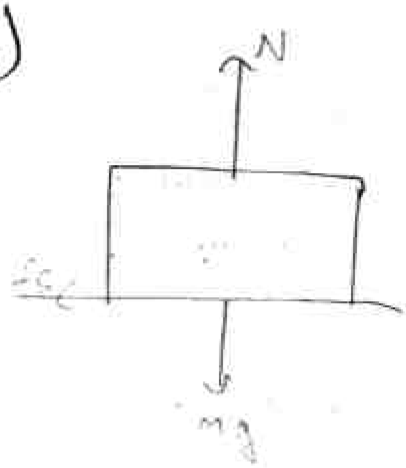
First law (static friction)

$f_s \propto N$

$f_s = M_s N$

$\frac{f_s}{N} = M_s$

$\boxed{\frac{f_s}{mg} = M_s}$



$\boxed{N = mg}$

M_s → co-efficient static friction.

∴ If f → Maximum.

$\boxed{M_s = \frac{f_2}{mg}}$

ii) kinetic friction.

→ Kinetic friction always acts in a direction ~~appositite~~ opposite to the direction in which the body is moving.

→ kinetic friction is $\perp$ to the normal ~~direction~~ reaction.

$$f_k \propto N$$

$$f_k = \mu_k N$$

$$\mu_k = \frac{f_k}{N}$$

$$\boxed{\mu_k = \frac{f_k}{mg}}$$

μ_k → coieffficient of kinetic friction.

iii) Rolling friction

→ Rolling friction increases with increases with. ~~Rolling~~ area of constant

→ Rolling friction is $\propto$ normal reaction.

→ Rolling friction inversly Propostional to the radias

$$f_R \propto \frac{N}{r}$$

$$f_R = \frac{\mu_R N}{r}$$

$$\mu_R = \frac{f_R r}{N}$$

$$\mu_R = \frac{f_R^2}{mg}$$

→ Determine the maximum acc of train which a box laying on its ~~floor~~ floor will remain stationary, Given that the ~~coeffcnt~~ co-efficient of static friction b/w the box & the train floor is 0.15

$f_s = \mu_s N$

$f_s = ma \quad N = mg$

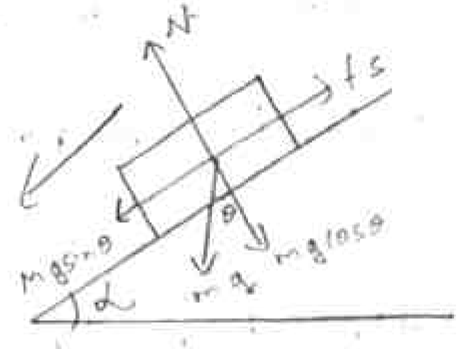
$\cancel{m} a = \mu_s \cancel{m} g$

$a_{max} = \mu_s g$

$= 0.15 \times 9.8$

$\boxed{a_{max} = 1.47 ms^{-2}}$

→ Free body diagram ~~for st~~ of an object sliding on a Inclined plane

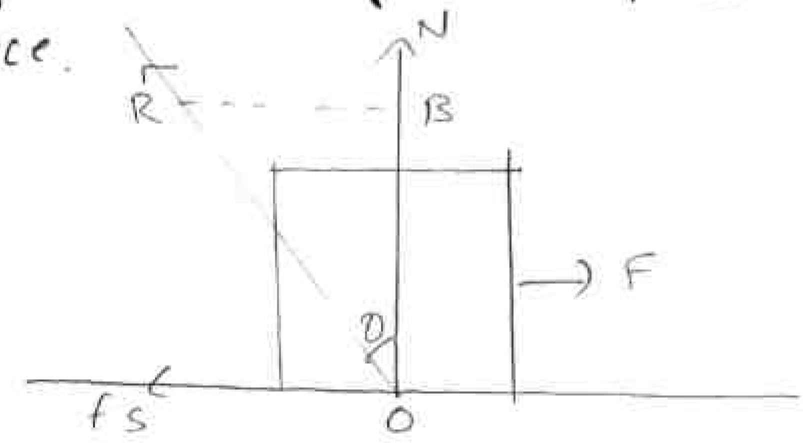


$N = mg \cos\theta$

$f_s = mg \sin\theta$

* Angle of friction.

angle subtended by Normal force and Resultant force.



The angle which the resultant of the limiting friction and the normal reaction is known as angle of friction.

$$\tan\theta = \frac{BR}{OB}$$

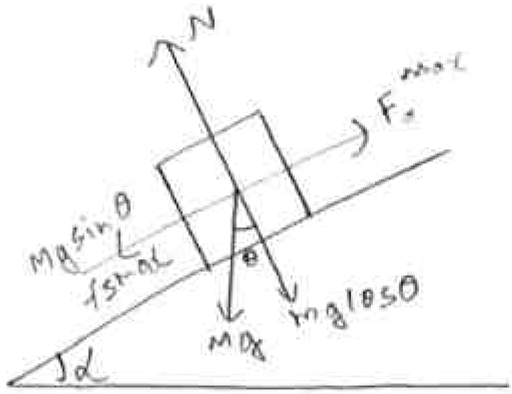
$$\boxed{\tan\theta = \frac{f_s^{max}}{N}}$$

$$\boxed{\tan\theta = \frac{f_L}{N}}$$

$$\boxed{\tan\theta \cong \mu_s}$$

$$\theta = \tan^{-1}(\mu_s)$$

* Angle of Repose



~~From Horiz~~

It is the minimum angle that an enclined plain makes with the horizontal when a body placed on it just begins to slide down.

From Horizontal forces

$$F_s^{max} = mg\sin\theta \rightarrow (1)$$

From vertical force

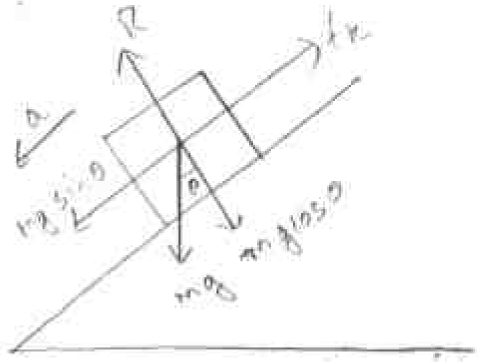
$$N = Mg\cos\theta \rightarrow (2)$$

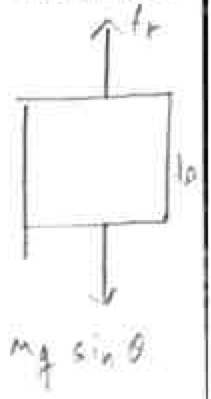
From eq (1) ÷ (2)

$$\frac{F_s^{max}}{N} = \frac{\cancel{Mg}\sin\theta}{\cancel{Mg}\cos\theta} \qquad . f = f_s^{max}$$

$$\boxed{\frac{f_s^{Max}}{N} = \tan\theta}$$

* Acc of the body sliding ~~down~~ on a rouf inclined plain





by applying Newton's 2^{nd} law

$F = ma$

$mg\sin\theta - f_k = ma$

$mg\sin\theta - \mu_k mg\cos\theta = ma$

$\cancel{m}g(\sin\theta - \mu_k \cos\theta) = \cancel{m}a$

$$\boxed{a = g(\sin\theta - \mu_k\cos\theta)}$$

$f_k = \mu_k R$

$\therefore R = mg\cos\theta$

$f_k = \mu_k mg\cos\theta$

* Meathods of reducing friction.

i) ~~Pollsin~~ Pollshing

ii) lubretants.

iii) Ball Bearing

iv) Anti friction Alloy.

v) Air kushens

* Meathods of increasing the friction.

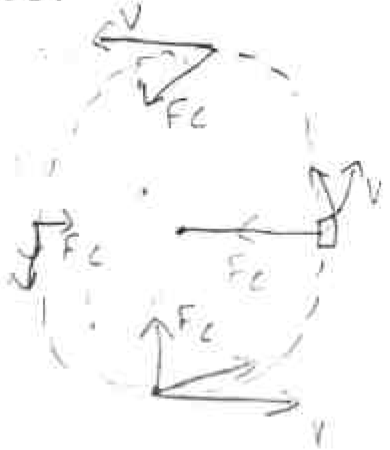
i) Treading of Tires. (Friction b/w road & tyers

ii) Sand is thrown on Railway tracks to increase the friction.

* Centripetal force.

~~It is a fo.~~

A force required to make a body move along a cirqular path with uniform speed is called as centripetal force.



for linear acc

$F_c = ma_c$

$$\boxed{F = \frac{mv^2}{r}}$$

$$\boxed{a_c = \frac{v^2}{r}}$$

Angular acc

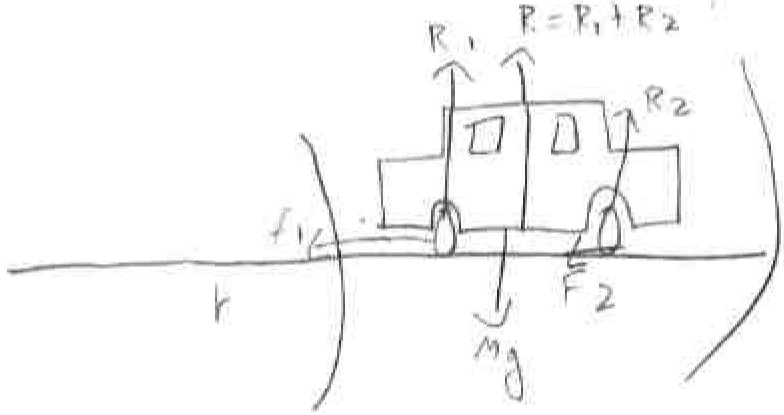
$$\boxed{F_c = mr\omega^2}$$

Eg: i) Stone rotated in a circle

ii) electron revolve around the neucleas

iii) cars taking a circular turn.

★ Circular motion of a car in level Road.



Consider A car moving on the level road the forces act on the car

i) the weight of the car = mg
ii) Normal reaction = N
iii) Frictional force = f

As there is no acc in the vertical direction. the centripetal force required for circular motion is along the surface of the road, and is provided by the component of the contact force b/w road & the car tyers along the surface.

The frictional force, total frictional force $= f_1 + f_2$

$= \mu_k R_1 + \mu_k R_2$

$= \mu_k (R_1 + R_2)$

$F = \mu_k mg$

From centrifctal force, $F = \frac{mv^2}{r}$

$F = \mu_k mg$

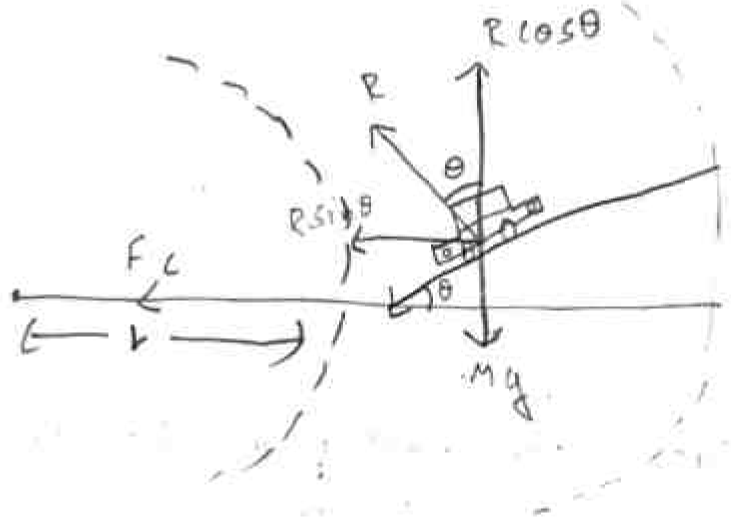
$\frac{mv^2}{r} = \mu_k Mg$

$v^2 = \mu_k g r$

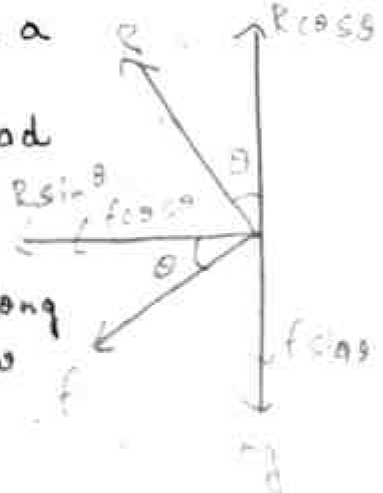
$\boxed{v = \sqrt{\mu g r}}$ $\mu \rightarrow$ Co-efficient friction

Case ii) A circular motion of a ~~car~~ car on a banked road

IMP



consider a car moving on a ~~a~~ circular road in in circular motion, If the road is Banked. Since there is no acc along a vertical direction, the net force along this direction must be zero



From vertical forces.

$R\cos\theta = Mg + f\sin\theta$

$R\cos\theta - f\sin\theta = Mg \rightarrow ①$

From Horizontal forces

$R\sin\theta + f\cos\theta = F_c$

centripetal force $= F_c = \frac{mv^2}{r}$

$R\sin\theta + f\cos\theta = \frac{mv^2}{r} \rightarrow ②$

From eq $\frac{②}{①}$

$$\frac{R\sin\theta + f\cos\theta}{R\cos\theta - f\sin\theta} = \frac{\frac{\cancel{M}v^2}{r}}{\frac{\cancel{M}g}{1}}$$

$$\left\{ \frac{\frac{v^2}{r}}{\frac{g}{1}} = \frac{v^2}{r} \times \frac{1}{g} \right.$$

$$\frac{R\sin\theta + f\cos\theta}{R\cos\theta - f\sin\theta} = \frac{v^2}{rg} \rightarrow ③$$

Note:- $N = R$ = Normal Reaction

friction

$f = \mu R$ or $f = \mu N$

$\frac{f}{R} = \mu \quad \frac{f}{N} = \mu$

devide eq ③ by $R\cos\theta$

$$\frac{\frac{\cancel{R}\sin\theta}{\cancel{R}\cos\theta} + \frac{f\cancel{\cos\theta}}{R\cancel{\cos\theta}}}{\frac{\cancel{R\cos\theta}}{\cancel{R\cos\theta}} - \frac{f\sin\theta}{R\cos\theta}} = \frac{v^2}{rg}$$

$$\frac{\tan\theta + \frac{f}{R}}{1 - \frac{f}{R}\tan\theta} = \frac{v^2}{rg}$$

$$\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{v^2}{rg}$$

$$v^2 = rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)$$

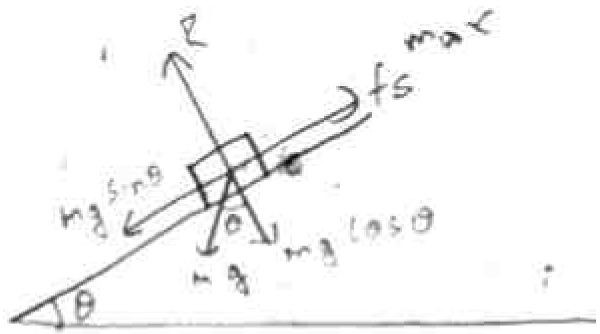
If $v = v_{max}$ and $\mu = 0$

$$v_{max}^2 = rg\left(\frac{\tan\theta + \cancel{(0)}}{1 - \cancel{(0)\times\tan\theta}}\right)$$

$$v_{max}^2 = rg\tan\theta$$

$$\boxed{v_{max} = \sqrt{rg\tan\theta}}$$

i) A mass of 4kg rests on a horizontal plane. the plane is gradually inclined at an angle 'θ' 15^0 with the Horizontal, the mass just ~~beg~~ begins to slide. What is the co-efficient of static friction b/w the block and the surface



$f_s^{max} = \cancel{m}g \sin\theta \rightarrow (1)$ { Horizontal

$R = \cancel{m}g \cos\theta \rightarrow (2)$

$f_s^{max} = \mu_s R$

$\mu_s = \dfrac{f_s^{max}}{R}$

$\dfrac{eq(1)}{eq(2)} = \tan\theta = \dfrac{f_s^{max}}{R}$

$\tan\theta = \mu_s$

$\mu_s = \tan(15)$

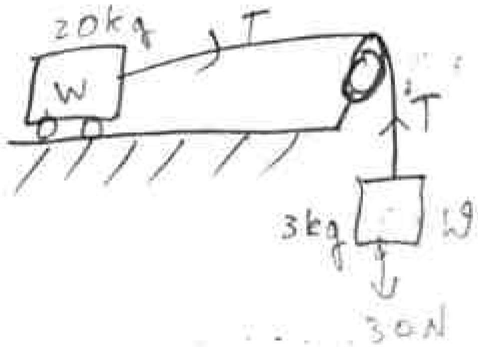
$\boxed{\mu_s = 0.27}$

$\tan 45^\circ = 1$

$\tan 30 = \dfrac{1}{\sqrt{3}} = \dfrac{0.57}{2}$ 28

$\tan 15 = 0.235$

ii) What the acc of Block and trolly system shown in fig, If the co-efficient of kinetic friction b/w the trolly and the surface is ~~0.04~~ 0.04 ? What is the tension in the ~~spring~~ string ? take $g = 10\ m/s^2$. Neglete the mass of the string.



$$T = \frac{m_2 m_1 g}{(m_1 + m_2)}$$

$$T = \frac{(20 \times 3) \times 10}{(3+20)} = \frac{60 \times 10}{23} = \frac{600}{23} = 26.08\ N$$

iii) A cyclist speeding at 18 km/h on a level road takes a ~~short~~ sharp circular turn ~~whio~~ of radius 3 m, without reducing speed the 10 co-efficent of staticfriction of ~~SF b/we~~ SF b/w the trees & ~~recod~~ ~~record~~ road is 0.1 will the cyclist slip while taking the turn.

$v^2_{max} \leq \mu r g$

$v = 18\, kmph$

$v_{max} \leq \sqrt{\mu r g}$

$= \cancel{18} \times \frac{5}{\cancel{18}}$

$v^2_{max} \leq 0.1 \times 3 \times 9.8$

$v = 5\, m/s$

$25\, ms^{-1} \leq 2.94\, ms^{-1}$

$v = 25\, ms^{-1}$

It does not obeys condition

iv) A cirqular race track of radias 300m is Banked ~~betan~~ at an angle of 15^0 If the co-efficient of fiction of a wheels of a race car and the road is 0.2, what is the

i) what is the optimum speed of the race car to avoid ~~where and te~~ wear & tear on it tyers, and

ii) Maximum permesible speed to avoid slipping

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$$v = \sqrt{rg \tan\theta}$$

$$= \sqrt{300 \times 9.8 \times \tan \times 15}$$

$$v = \sqrt{300 \times 9.8 \times 0.27}$$

$$v = 28.17 \text{ m/s}$$

$$V_{max} = \sqrt{rg \frac{\tan\theta - \mu}{1 - \mu \tan\theta}}$$

$$= \sqrt{\frac{300 \times 9.8 \times (0.27 - 0.2)}{(1 - 0.2 \times 0.27)}}$$

$$= \sqrt{\frac{2940 (0.07)}{0.046}}$$

$$= \sqrt{}$$