chapter

Work, Energy and Power

Work :-

Work is defined force × displacement

Work is said to be done by a force if point of application of force undergoes displacement either in the direction of force along the component of force.

Consider a particle which moves from one point to another.

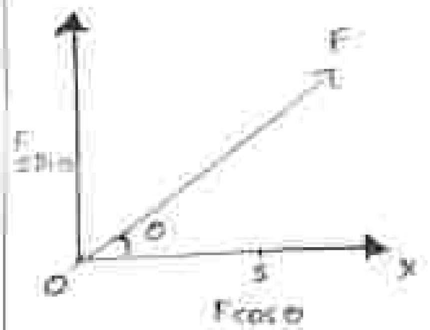
let 'F' be the force acting on it

let 's' be the displacement along the line of action it

cause angle between force and displacement

Work done = F× displacement along the line of action

$$W = \vec{F} \cdot S$$



$W = F\cos\theta \cdot S$

$\therefore W = Fs\cos\theta$

[scalar dot product from vectors]

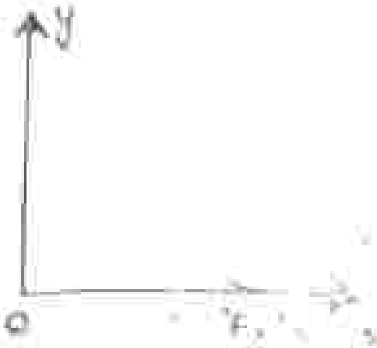
SI unit of work is Joule, Nm, kgm^2s^{-2}

Dimensional formula :- $[M^1L^2T^{-2}]$

Work is a scalar quantity

Nature of work [work may be positive or negative]

Case 1 -



$W = FS\cos\theta$

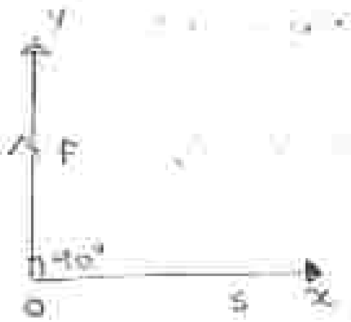
when $\theta = 0°$ [Parallel]

$W = F.S\cos$ [0]

$W = F.S$

work is positive

Case II



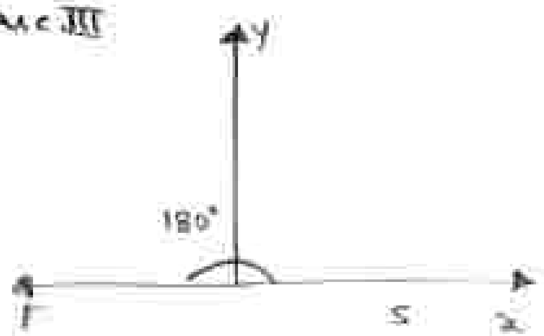
when $\theta = 90$ ($\perp^{r}$)

$W = FS\cos$ (90)

$F.S$ (0)

$W = 0$

Case III



$W = F\cos(180°)$

$F.S(-1)$

$W = -F.S$

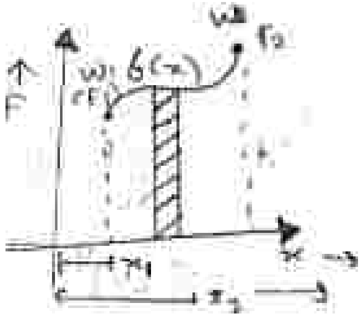
work done is negative

Work done by variable force.

If a particle is subject varying a force "F" but maintain a constant direction the workdone in moving the body from $x_1 \to x_2$ is

$$dw = \int_{x_1}^{x_2} F \cdot dx$$

$$\Delta W = F \Delta S$$



Total work done

$$W = \Sigma \Delta W = \Sigma F \Delta S$$

$$W = \lim_{\Delta s \to 0} F \cdot \Delta S$$

Magnitude and Direction.

$$W = F(x)\, dx$$

$$F = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} \rightarrow \text{Force}$$

$$x = dx \hat{i} + dy \hat{j} + dz \hat{k} \rightarrow \text{Displacement}$$

Total workdone in magnitude and Direction.

$$dW = F_x dx + F_y dy + F_z dz$$

$$\frac{dv}{dx} = 0 = W$$

$$dw = F(x) \cdot dx$$

Integrate on both sides

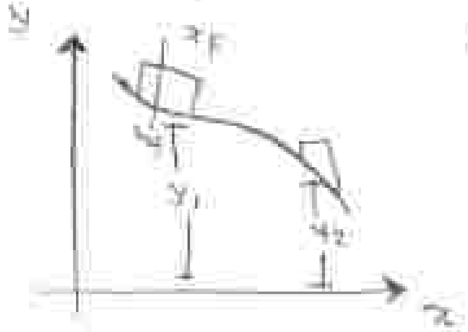
$$\int_0^W dw = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$$

$$W = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$$

* Work done

Force of gravity

let us consider a body of mass 'm' moving from one place to another place

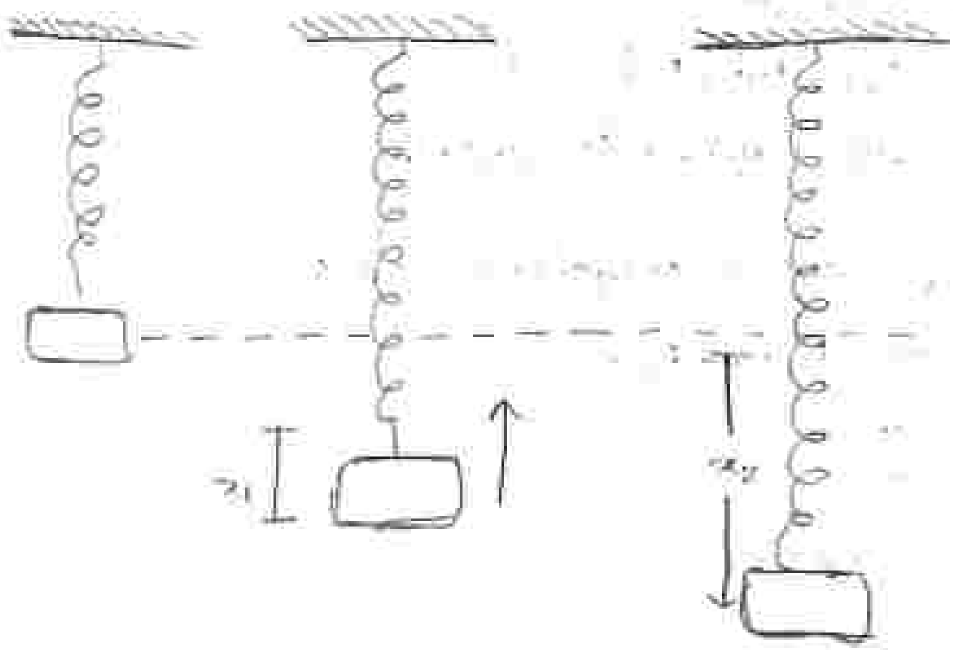


$$x_1 \rightarrow x_2 \quad \int_0^w dw = \int_{y_1}^{y_2} F \cdot dy$$

$$w = F[y]^{y_2}$$

$$W = -mg[y_1 - y_2]$$

Work done by the spring



Total work done by the spring

$$dw = F \cdot dx$$

$$F = -kx$$

$$\int_0^w dw = \int_{x_1}^{x_2} F \cdot dx$$

$$W = \int_{x_1}^{x_2} -kx \cdot dx$$

$$W = -k \int_{x_1}^{x_2} x \cdot dx$$

$$= -k\left[\frac{x^2}{2}\right]_{x_1}^{x_2}$$

$$= -k\left[\frac{x_2^2}{2} - \frac{x_1^2}{2}\right]$$

$$W = -\frac{kx_2^2}{2} + \frac{kx_1^2}{2}$$

$$W = \frac{kx_1^2}{2} - \frac{kx_2^2}{2}$$

Note :- workdone by a spring force can be ~~neg~~ positive negative or zero

Work and ~~kinetic~~ energy theorm [work and kinetic energy]

Under rectilinear motion constant acceleration a has been encountered

Work energy theorm.

From kinetic eq^n (3)

$$v^2 - u^2 = 2as$$

Multiply $\frac{1}{2}m$ on both sides

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = 2 \times \frac{1}{2} \times m \times as$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = F.s$$

$$KE_f - KE_i = F.s$$

$$KE_f - KE_i = W$$

$$\Delta KE = W$$

where KE_i and KE_f and final kinetic energy for the object work done by a force on the body over a certain displacement

Statement :- The change in kinetic energy of a particle is equal to the work done on it by the net force

Work done by the internal force [Internal work]

$$W = \Delta KE$$

$$W_{internal} + W_{external} = \Delta KE$$

An object is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j})$ m to $r_2 = (4\hat{i} + 6\hat{j})$ m under a force $\vec{F} = (3x^2\hat{i} + 2y\hat{j})$. Find the work done by this force.

$$W = F.dx$$

$$W = (3x^2 + 2y) . dx$$

$$W = \int (3x^2 + 2y) . dx$$

$$\int_{(2,3)}^{(4,6)} 3x^2 . dx + \int 2y . dy$$

$$\int_{2}^{4} 3x^2 . dx + \int_{3}^{6} 2y . dy$$

$$\left[\frac{3x^3}{3}\right]_2^4 + \left[\frac{2y^2}{2}\right]_3^6$$

$$\left[x^3\right]_2^4 + \left[y^2\right]_3^6$$

$$= \left[4^3 - 2^3\right] + \left[6^2 - 3^2\right]$$

$$= \left[64 - 8\right] + \left[36 - 9\right]$$

$$= \left[56 + 27\right]$$

$W = 83\ J$

Application-1

Work done ~~in~~ by a gas in a cylinder by a friction

$dW = F.dy$ $\qquad$ $A.dy = dV$

$dW = PA.dy$

$V_1 \to V_2$

$$\int_0^W dW = \int_{V_1}^{V_2} P.dV$$

$$W = P(V_2 - V_1)$$

Application 2:

The uniform change of mass m and length L rest on a table having $\frac{1}{nm}$ part of its length, hanging down from edge of a table the work done by the pulling force to bring the hanging part of chain on to the table

$W = F \times h$

$= mg \times h$

$\frac{m}{n} g \times \frac{l}{2n}$

$W = \frac{mgl}{2n^2}$

A massive box is dragged along a horizontal floor by a rope. The rope makes an angle of 60° with the horizontal. Find the work done the tension in the rope is 200N and the box is moved through the distance of 20m

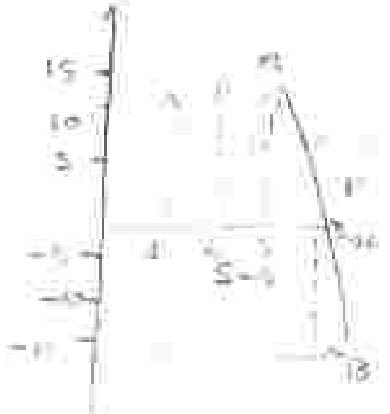
$\Rightarrow W = F\cos\theta$

$200 \times 20 \times \cos(60^\circ)$

$\cancel{4000}^{2000} \times \frac{1}{\cancel{2}}$

$W = 2000J$

A body ~~[illegible]~~ moves from point A to B under the action of force ~~[illegible]~~ varying in magnitude as shown in the figure. Obtain the work done force is expressed in newton and displacement in meter



Total workdone $W_{AP}+W_{PQ}+W_{QR}+W_{RB}$.

$W_{AP} = 10\times1$
$= 10J$

$W_{PQ} = \frac{1}{2}[15+10]$
$= 12.5(J)$

$W_{QR} = \frac{1}{2}(15+0)$
$= 7.5$

$W_{RB} = \frac{1}{2}(-15) = -7.5J$

$= 10+12.5+\cancel{7.5}-\cancel{7.5}$

$W = 22.5J$

Power $= \frac{W}{t}$

$P = \frac{F.x}{t}$

$\frac{dp}{dt} = \frac{d}{dt}(F.x)$

$P = F.\frac{dx}{dt}$

$P = F.V$

$\frac{dp}{dt} = \frac{dw}{dt}$

$\therefore$ Integration on both sides

$P = \lim_{\Delta t\to0} \frac{W}{t}$

$P = \lim_{\Delta t\to0} \frac{dw}{dt}$

$P = \frac{\Delta W}{\Delta t}$ $\rightarrow$ SI unit watt

1 hp = 735 watts

Power is a rate of doing work

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

$$P = \frac{F.x}{t}$$

SI unit of Power : Watt or Joule per second

We can express the unit of power in HP

1 watt = 1 Joule per second

1 HP = 746 watt

Power is a scalar quantity

Dimensional formula for power = $[M^1L^2T^{-3}]$

Efficiency of power is = $\eta = \frac{\text{Output power}}{\text{Input power}}$

Sand drop from a stationary hopper at the rate of 5kg per second on to a conveyer belt with constant speed of two metre per second what is the force required to keep the belt moving and what is the power delivered by the motor moving the belt

$\Rightarrow P = \frac{W}{t} = \frac{FS}{t}$

$= 10 \times 2 = 20\text{watt}$

Energy:

Example: An engine of power 1.54MW applies a force of 6 minutes on a train moving with a velocity of $10ms^{-1}$ If there is no friction and the velocity attained is $25ms^{-1}$. Find the mass of the train.

Sol: Power = 1.54 MW = $1.54 \times 10^6 w$

Time 60min => 360sec

Final velocity v = 25m/s

Mass of the train = m

$$\text{Power} = \frac{w}{t} = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mu^2}{t}$$

$$= 1.5 \times 10^6$$

$$= \frac{\frac{1}{2}m[25^2 - 10^2]}{360}$$

$$m = \frac{1.5 \times 10^6 \times 2 \times 360}{625 - 100} = 2.05 \times 10^6 kg$$

Energy: The capacity to do work is called energy.

* Energy is required for doing work
* Energy is a Scalar quantity
* SI unit of energy is Joule
* Dimensional formula of energy $[M^1L^2T^{-2}]$

Forms of energy.

* Sound energy • Mechanical energy, heat energy, chemical energy, Chemical energy • nuclear energy

Types of Energy [Mechanical Energy]

Mechanical energy of two types

1. Kinetic energy : A particle in motion is having kinetic energy because of its velocity • It is denoted by K

$K = \frac{1}{2} mv^2$

SI unit of kinetic energy is Joule or Newton metre [Nm]

Kinetic energy is always positive

consider the work done by moving object, consider a bullet of mass m moving with velocity v which strikes a wooden plank and penetrates through a distance x before coming to rest

Initial velocity of bullet $u = v$

Final velocity of Bullet $V = 0$

Distance travelled $= x$

$S = x$

By using kinematic equation

$V^2 - u^2 = 2as$

$0 - u^2 = 2 \times a \times x$

$a = \frac{-u^2}{2x}$

From newton's 2nd law

$F = ma$

$F = m\left[\frac{-u^2}{2x}\right]$

work done by the bullet $= -Fs$

$W = -Fx$

$W = -Fx$

$= -m\left(\frac{u^2}{2x}\right)x$

$W = \frac{mu^2}{2}$

$W = \frac{1}{2}mu^2$

$W = \Delta KE$

$KE = \frac{1}{2}mv^2$

Potential energy defined as work done by the object from certain height from virtue of its motion

$P = mgh$

workdone = Force x Displacement

$W = F \times S$

$F = Mg$ [gravitational force]

S [Displacement] [h]

$W = mgh$

$W =$ Potential energy.

Conservative force

Conservative forces

In conservative forces.

(1) Conservative force

$W_{closed\ path} = 0$

Independent on Nature

(2) Non-conservative force

$W_{closed\ path} \neq 0$

Dependent on Nature

It is Independent of the path followed by the particle

Ex: Gravitational force

Elastic force

$$F_{conservative} = \frac{\partial u}{\partial x}$$

$$F_{conservative} = -\frac{\partial u}{\partial x}\hat{i} - \frac{\partial u}{\partial y}\hat{j} - \frac{\partial u}{\partial z}\hat{k}$$

Non conservative :

If the workdone by the force around a closed path it is not equal to zero and it is dependent on the path such forces are called non-conservative forces

eg. Kinetic friction

Viscous force

Relation between K.E linear momentum

Kinetic energy $= \frac{1}{2}mv^2 \Rightarrow$ K.E

linear momentum $mv = P$

$$K.E = \frac{1}{2}mv^2$$

$k.E = \frac{1}{2} m \frac{P^2}{m^2}$

$k.E = \frac{1}{2} \frac{P^2}{m}$ k → kinetic energy

m → mass

$P = \sqrt{2mk}$ → interms of momentum

$k.E = \frac{P^2}{2m}$ P → momentum

If $V = 3\hat{i} + 4\hat{j} + 5\hat{k}$ is the instantaneous velocity of the body of mass 1.50 kg. Calculate its kinetic energy

⇒ kinetic energy $= \frac{1}{2} mv^2$

$= \frac{1}{2} \times 1.5 \times (3\hat{i} + 4\hat{j} + 5\hat{k})(3\hat{i} + 4\hat{j} + 5\hat{k})$

$= 7.5(9 + 16 + 25)$

$= 0.7.5(50)$

$K.E = \cancel{3750}\ 0.75(50)$

$k.E = 37.5\,J$

A cyclotron accelerates a proton to a final speed 3×10^7 m/s which is initial at rest. Find how much is done on the proton by the electrical force of the cyclotron. Mass of the proton is.

⇒ $M_p = 1.67 \times 10^{-27}$ kg

$V = 3 \times 10^7$ m/s.

$k.E = W$

$k.E = \frac{1}{2} mv^2$

$= \frac{1}{2} \times 1.67 \times 10^{-27} \times (3 \times 10^{7})^{2}$

$= 1.67 \times 10^{-13} \times 45$

$= 7.515 \times 10^{-13} J$

$W = 7.515 \times 10^{-13} J$

Conservation of Mechanical energy

The sum of kinetic energy and potential energy of a particle remains constant when it is subjected to conservative force

$W_{1 2} = U_1 - U_2$

$W_{1 2} = K_1 - K_2$

$U_1 - U_2 = K_1 - K_2$

$\underbrace{U_1 + K_1} = K_2 + U_2$

$PE + KE = PE + KE$

$KE + PE = constant$

Total energy $= KE + PE$

In a balance stick demonstration a police officer fires a bullet of 50 g bullet with speed 20 m/s on soft plywood of thickness 2 cm the bullet emerges with only 10% of its initial kinetic energy. what is the emergent speed of the bullet is [Kinetic energy of bullet 1000J]

$\Rightarrow$ K.E = 1000J [Bullet]

10% KE = 1000 − 900 = 100J

100 − 10 = 90%

$$K.E = \frac{1}{2} m v_f^2$$

$$100 = \frac{1}{2} \times 0.05 \times v_f^2$$

$$v_f = \sqrt{\frac{100 \times 2}{0.05}}$$

$$v_f = \sqrt{\frac{200}{0.05}}$$

$$v_f = \sqrt{4000}$$

$$v_f = 63.24$$

Alternative Units of work / Energy in J

erg	10^{-7} J
Electron volt (ev)	1.6×10^{-19} J
Calorie (cal)	4.186 J
Kilowatt hour (kwh)	3.6×10^6 J

Typical kinetic energies

Object	Mass	Speed	K
Car	2000	25	6.3×10^5
Running Athlete	70	10	35×10^3
Bullet	5×10^{-2}	200	10^3
Stone dropped from 10m	1	14	10^2 J
Rain drop at terminal speed	3×10^{-5}	9	1.4×10^{-3}
Air molecule	$\simeq 10^{-26}$	500	$\simeq 10^{-21}$

A cyclist come to skating top in 10m. During this process the force on cycle due to road is 200N and it is directly opposed to the motion.

(1) How much work does the road do on the cycle

(2) How much work does the cycle do on the road

$W = F \cdot d$

$W = F \cdot d \cos\theta$

$= 200 \times 10 \times \cos(180^\circ)$ or 180°

$= 2000(-1)$

$W = 2000 J$

Example 5.2 : It was is well known that a raindrop falls under the influence of the downward gravitational force and the opposing resistive force. The latter is known to be proportional to the speed of the drop but is otherwise undetermined. Consider a drop of mass 1.00g falling from a height 1.00km. It hits the ground with a speed of 50.0 ms^{-1}

(A) what is the work done by the gravitational force? what is the work done by the unknown resistive force.

(a) $\Rightarrow \Delta K = \frac{1}{2} mv^2$

$\frac{1}{2} \times 10^{-3} \times 50 \times 50$

$= 1.25 J$

$W_g = mgh$

$= 10^{-3} \times 10 \times 10^{3}$

$= 10\,J$

Work done by the resistive force

$W_R = \Delta k - W_g$

$= 1.25 - 10$

$= -8.75\,J$

To stimulate car accidents, auto manufactures steady the collision of moving cars with mounted springs of different spring constants. Consider a typical simulation with a car of mass 1000kg moving with a speed 18km/h has a smooth road and colliding with a horizontally mounted spring of spring constant $5.25 \times 10^{3} Nm^{-1}$ what is the maximum compression of the spring

$\Rightarrow K = \frac{1}{2} mv^2$

$= \frac{1}{2} \times 10^{3} \times 5 \times 5$

$= \frac{1}{2} \times 10^{3} \times 25$

$= 1.25 \times 10^{4}\,J$

Equilibrium

Any body - the net force on it must be zero

Types of equilibrium

(1) Stable equilibrium

(2) Unstable equilibrium

(3) Neutral equilibrium

Stable equilibrium : when a particle is displaced slightly from its initial equilibrium position, if the force acting on it brings the particle back to initial position is called stable equilibrium

Unstable equilibrium. when a particle is displaced slightly from its initial equilibrium position, if the force acting on it tends to displace the particle further away from initial position, it is called unstable equilibrium

Here potential energy is maximum.

Neutral equilibrium

when a particle is slightly displaced from its initial equilibrium positions it continuous to remain in the displaced position it is said to be in neutral equilibrium

Here potential energy is constant

$$\frac{d^2U}{dx^2} = 0$$

law of conservation of energy.

Under the action of conservative force and no work is done by the external forces total mechanical energy of the system remains constant

Total Energy = Kinetic energy + Potential energy

$$K + U = 0$$

$$K = -U$$

$$\Delta KE = -\Delta U$$

Change in kinetic energy

change in potential energy

law of conservation of energy in case of freely falling body

A ball of mass "m" is dropped from a certain height "h". The total energy of the ball TE is $PE = KE + U$ where KE is a kinetic energy u is the potential energy

At point A

$TE = KE + PE$

$KE = 0$ {there no motion}

$PE = mgh$

h → height of an object

$TE = 0 + mgh$

mg → gravitational force

$TE = mgh \rightarrow (1)$

At point "B"

$TE = KE + PE$

$KE = \frac{1}{2} mv^2$

From kinetic eqn $u = 0$

$v^2 - u^2 = 2as$ $a = g$

$v^2 = 2g(h-x)$ $s = h-x$

$KE = \frac{1}{2} m2g(h-x)$

$KE = mg(h-x)$

$PE = mas$

$PE = mgx$

$TE = KE + PE$

$mg(h-x) + mgx$

$mgh - mgx + mgx$

$TE = mgh$

At point C

$TE = KE + PE$

$KE = \frac{1}{2}mv^2$

$v^2 - u^2 = 2as$

$v^2 = 2gh$

$KE = \frac{1}{\cancel{2}} m \cancel{2}gh$

$KE = mgh$

$PE = mg \times 0$

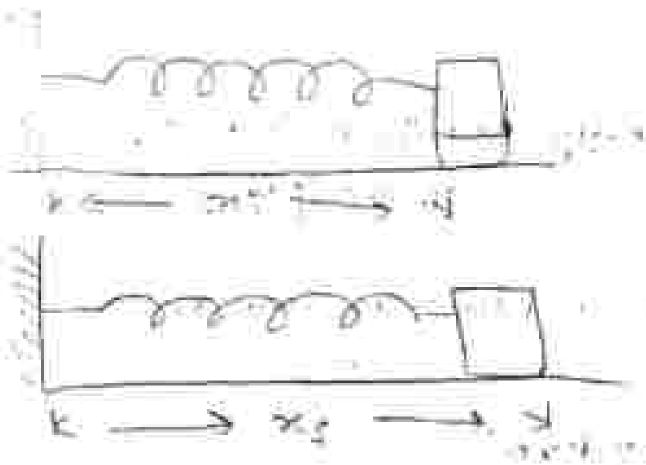
$PE = 0$

$TE = KE + PE$

$mgh + 0$

$\underline{TE = mgh}$

Potential energy of the spring



Consider a spring constant k and initial displacement x_i and final displacement x_f then the work done by the spring force is

$$dw = F \cdot dx \rightarrow (1)$$

$$F_s = -kx$$

$$dw = -kx\,dx$$

Integrate eq w.r.t. x and from x_i to x_f

$$\int_0^w dw = \int_{x_i}^{x_f} -kx\,dx$$

$$w = -k\int_{x_i}^{x_f} x\,dx$$

$$-k\left[\frac{x^2}{2}\right]$$

$$-k\left[\frac{x_f^2}{2} - \frac{x_i^2}{2}\right]$$

$$= -\frac{kx_f^2}{2} + \frac{kx_i^2}{2}$$

$$w = \frac{1kx_i^2}{2} - \frac{1}{2}kx_f^2$$

$$w = \frac{1}{2}kx_m^2$$

Total mechanical energy Maximum velocity of the spring

PE = KE

$w = U = V(x)$ in b/w $-x_m$ and x_m

$x = 0$

$$V(x) = \frac{1}{2}kx_m^2$$

$$V(x) = \frac{1}{2}k(0)^2 = 0$$

Total mechanical energy

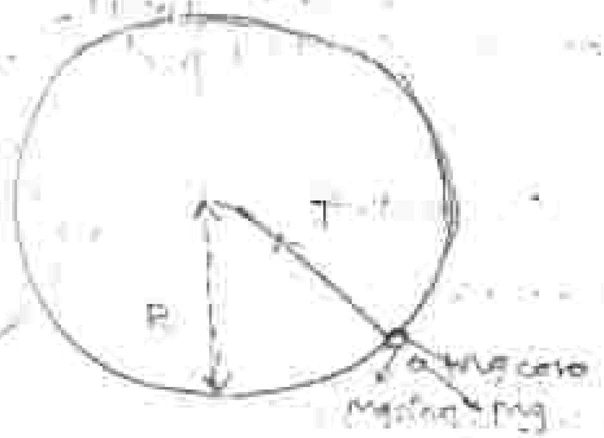
$$\frac{1}{2}kx_m^2 - \frac{1}{2}kx_m^2 = \frac{1}{2}mv^2$$

$$\frac{1}{2}kx_m^2 = \cancel{\frac{1}{2}kx^2}^{\,0} + \frac{1}{2}mv_m^2$$

$$\cancel{\frac{1}{2}}kx_m^2 = \cancel{\frac{1}{2}}mv_m^2$$

$$v_m^2 = \frac{k}{m}x_m^2$$

$$v_m = \left(\sqrt{\frac{k}{m}}\right)x_m$$



When circular motion is performed in a plane having gravity along the plane then it is called vertical circular motion.

Consider a particle of mass having gravity and it is attached inextensible light spring string of length R the particle is moving in a vertical circle of radius R about a fixed point

From newtons second law

$F = ma$

The tension acting towards centre

$F = T - mg\cos\theta \rightarrow (1)$

Centripetal force $F_c = \frac{mv^2}{r} \rightarrow (2)$

equate (1) and (2)

$\frac{mv^2}{R} = T - mg\cos\theta$

$T = \frac{mv^2}{R} + mg\cos\theta \rightarrow (1)$

For minimum velocity of the object at vertical direction

$T = 0 \quad \theta = 180°$

$T = \frac{mv^2}{R} + mg\cos(180°)$

$0 = \frac{mv^2}{R} + mg(-1)$

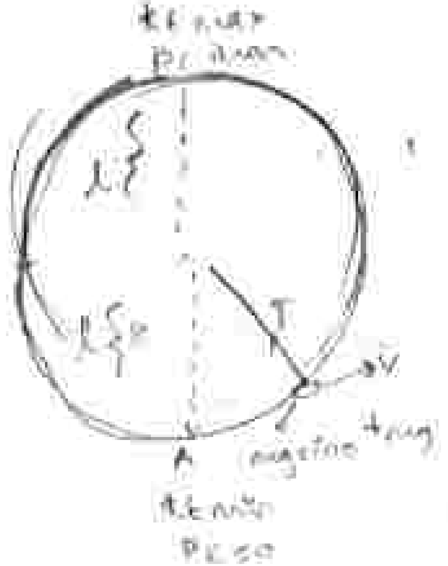
$0 = \frac{mv^2}{R} - mg$

$V = V_m$ { minimum velocity for the object for vertical motion

$\frac{MV^2}{R} = Mg$

$V_m^2 = gr$

$V_m = \sqrt{gr}$



From law of conservation

At point A = KE = minimum = $\frac{1}{2}mv_m^2$

$PE = mg(0) = 0$

At point B = $KE = \frac{1}{2}mv^2$

$PE = mgh \{h = 2l$

$= mg2l$

At point A = at point B

$PE + KE = KE + PE$

$0 + \frac{1}{2}mv_m^2 = \frac{1}{2}mv^2 + 2mgl$

$m\left(\frac{1}{2}v_m^2\right) = m\left(\frac{1}{2}v^2 + 2gl\right)$

$\frac{1}{2}v_m^2 = \frac{1}{2}gR + 2gl$

$\frac{1}{2}v_m^2 = \frac{1}{2}gl + 2gl$

$\frac{1}{2}v_m^2 = \frac{5}{2}gl$ $\quad \left\{ = \frac{gl + 4gl}{2} \right.$

$v_m = \sqrt{5gl}$ $\quad = \frac{5gl}{2}$

For highest point

Collision :

A strong interaction between two bodies that occurs for a very short interval during which redistribution of momentum ignoring the effect of other forces is called collision

Types of collision [on the basis of law of conservation of Kinetic energy]

- Elastic collision
- Inelastic collision

In a collision kinetic energy of the system is equal to kinetic energy before collision

The collision is said to be perfectly elastic

Kinetic energy is conserved total energy is conserved

Eg: Collision between atomic particles
Collision between α particles with nucleus

Inelastic collision

In a collision kinetic energy of the system is not equal to kinetic energy before collision

Eg: Collision between two Automobiles in road
collision between two billiard balls

Classification of collision based on direction classification of collision based on direction

(1) Head on collision [1 dimensional collision)

(2) oblique collision

(1) Head collision : In a collision if motion of colliding particles before and after the collision are along the same straight line. then the collision is said to be Head on or 1 dimensional collision

(2) oblique collision : In a collision. If the motion of colliding particles before and after the collision not along the initial line of motion, then a collision is said to be oblique collision [3d collision]

Perfectly elastic head on collision between two bodies

Before collision

During collision

let, we consider head on collision between two particles
suppose u_1, u_2 and v_1, v_2 the respective velocity
before and after collision as shown in the above

Applying law of conservation momentum

(momentum) before collision = After collision (momentum)

$$P_1 + P_2 = P_1 + P_2$$

[Initial velocity] [for final velocity]

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \rightarrow (1)$$

Applying law of conservation

KE of before collision = KE of after collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\frac{1}{2}(m_1u_1^2 + m_2u_2^2) = \frac{1}{2}(m_1v_1^2 + m_2v_2^2)$$

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

$$m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1+v_1)(u_1-v_1) = m(v_2+u_2)(v_2-u_2) \rightarrow (2)$$

From eq^n $\frac{1}{2}$

$$\frac{m_1(u_1+v_1)}{m_1(u_1+v_1)(u_1-v_1)} = \frac{m_2(v_2-u_2)}{m_2(v_2+u_2)(v_2-u_2)}$$

$$\frac{1}{(u_1+v_1)} = \frac{1}{(v_2+u_2)}$$

$$V_2 + u_2 = u_1 + v_1$$

$$V_2 = u_1 + v_1 - u_2 \rightarrow (3)$$

This is the velocity after collision (2nd object)

$$V_1 = V_2 + u_2 - u_1 \rightarrow (4)$$

This is the velocity for after collision (1st object)

Replace eq(3) in eq(1)

$$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

$$m_1(u_1 - v_1) = m_2(u_1 + v_1 - u_2) - u_2$$

$$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - m_2u_2 - m_2u_2$$

$$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - 2m_2u_2$$

$$m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$$

$$u_1(m_1 - m_2) + 2m_2u_2 = (m_1 + m_2)V_1$$

$$V_1 = \frac{u_1(m_1 - m_2) + 2m_2u_2}{m_1 + m_2}$$

Replace eq^n (4) in eq^n (1)

$$m_1u_1 - m_1(V_2 + u_2 - u_1) = m_2V_2 - m_2u_2$$

$$m_1u_1 - m_1V_2 + m_1u_2 + m_1u_1 = m_2V_2 - m_2u_2$$

$$2m_1u_1 - m_1V_2 + m_1u_2 = m_2V_2 - m_2u_2$$

$$2m_1u_1 + m_1u_2 + m_2u_2 = m_1V_2 + m_2V_2$$

$$2m_1u_1 + u_2(m_1 - m_2) = (m_1 + m_2)V_2$$

$$V_2 = \frac{2m_1u_1 + u_2(m_1 - m_2)}{(m_1 + m_2)}$$

Case I

when $m_1 = m_2 = m$ then

$$V_1 = \frac{u_1(m_1 - m) + 2mu_2}{m+m}$$

$$V_1 = \frac{2mu_2}{2m}$$

$$V_1 = u_2$$

$$V_2 = \frac{2mu_1 + u_2(m - m)}{2m}$$

$$V_2 = \frac{2mu_1}{2m}$$

$$V_2 = u_1$$

when $u_2 = 0$ the second body is initially at rest

If $u_2 = 0$ $= \frac{u_1(m_1 - m_2)}{m_1 + m_2}$

$$V_2 = \frac{2m_1u_1}{m_1 + m_2}$$

Coefficient of restitution [e]

$$e^{\text{ratio}} = \frac{\text{relative velocity of separation after collision}}{\text{relative velocity of approach before collision}}$$

$$u_1 + V_1 = u_2 + V_2$$

From above eq^n

$$e = \frac{V_2 - V_1}{u_1 - u_2}$$

$$u_1 - u_2 = v_2 - v_1$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

Perfectly inelastic head on collision:

In a collision if the body is stick together or one gets embedded into other and move with common velocity after collision, it is called a perfect inelastic collision.

Applying conservation of Momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 + m_2u_2 = (m_1+m_2)v \rightarrow (1)$$

From conservation of energy

$$V = \frac{(m_1u_1 + m_2u_2)}{(m_1+m_2)}$$

$$KE = \frac{1}{2} m_1u_1^2$$

$$KE = \frac{1}{2} m_2u_2^2$$

$$\Delta KE = \frac{1}{2}(m_1+m_2)v^2$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}(m_1+m_2)v^2$$

Change in k.E

$$\Delta KE = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1+m_2)v^2$$

$$\Delta KE = \frac{1}{2}\left(m_1u_1^2 + m_2u_2^2 - (m_1+m_2)\left(\frac{m_1u_1 + m_2u_2}{m_1+m_2}\right)^2\right.$$

$$\frac{1}{2}\left(m_1u_1^2 + m_2u_2^2 - (m_1+m_2)\frac{(m_1u_1+m_2u_2)^2}{(m_1+m_2)}\right.$$

$$\Delta KE = \frac{1}{2}m_1u_1^2 + m_2u_2^2 = \frac{(m_1u_1+m_2u_2)^2}{m_1+m_2}$$

collision in 2 dimension.

Elastic collision 2D

Consider a body a mass m_1 and m_2, a second object m_2 is on rest first object moving with a velocity u_1 and colliding with 2^{nd} object in 2D it will give elastic collision, for two objects the plane determined by the final velocity direction of m_1 and m_2 in xy plane

The conservation of linear momentum implies that the entire collision is in xy plane the xy components equation are

For Horizontal collision

$$m_1u_1 + m_2(0) = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2$$

$$m_1u_1 = m_1v_1\cos\theta + m_2v_2\cos\theta \rightarrow (1)$$

For vertical collision

$$0 = m_1 v_1 \sin\alpha_1 + (m_2 v_2 \sin\alpha_2)$$

$$0 = m_1 v_1 \sin\theta_1 + - m_2 v_2 \sin\theta_2$$

Conservation of Mechanical energy

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 (0)^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} (m_1 v_1^2 + m_2 v_2^2)$$

$$m_1 u_1^2 = m_1 v_1^2 + m_2 v_2^2 \rightarrow (3)$$

Physical quantity	Symbol	Dimension	Units	Remarks
Work	W	$[ML^2T^{-2}]$	J	$W = F \cdot d$
Kinetic energy	K	$[ML^2T^{-2}]$	J	$K = \frac{1}{2} mv^2$
Potential energy	$V(x)$	$[ML^2T^{-2}]$	J	$F(x) = \frac{dv(x)}{dx}$
Mechanical energy	E	$[ML^2T^{-2}]$	J	$E = K + V$
Spring constant	k	$[MT^{-2}]$	Nm^{-1}	$V(x) = \frac{1}{2} kx^2$
Power	P	$[ML^2T^{-3}]$	W	$P = \frac{dW}{dt}$

Example 5.10

An elevator can carry a maximum load of 1800kg is moving up with a constant speed of $2ms^{-1}$. The frictional force opposing the force is 4000N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse power

⇒ The downward force on the elevator is

$F = mg + f_f$

$= (1800 \times 10) + 4000$

$= 22000\, N$

The motor must supply enough power to balance this force. Hence

$P = F \times v$

22000×2

$= 44000\, W$

$= 59\, hp$

Problem : 2 : A particle of mass m_1 makes a head on elastic collision with a stationary particle of mass m_2 what fraction of kinetic energy is

(a) Retained by m_1 ?

(b) Transferred to m_2

$$\Rightarrow v_1 = \frac{(m_1 - m_2)u + 2m_2(0)}{m_1 + m_2}$$

$$\Rightarrow \left[\frac{v_1}{u}\right]^2 = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] \quad \frac{\frac{1}{2} m_1 v_1^2}{\frac{1}{2} m_1 v^2} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right]$$

$$\frac{\text{K.E of } 1^{st} \text{ particle finally}}{\text{K.E of } 1^{st} \text{ particle initially}} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right]^2$$

∴ Fraction of K.E retained by 1^{st} body $\left[\frac{m_1 - m_2}{m_1 + m_2}\right]$

(b) $V_2 = \frac{2m_1u + (m_2 - m_1)0}{m_1+m_2} \Rightarrow \frac{V_2}{u} = \frac{2m_1}{m_1+m_2}$

$$f_2 = \frac{\frac{1}{2}m_2v_2^2}{\frac{1}{2}m_1u^2} = \frac{m_2}{m_1}\left[\frac{2m_1}{m_1+m_2}\right]^2$$

$$\therefore f_2 = \frac{4m_1m_2}{(m_1+m_2)^2}$$

Problem: 2 - Two perfectly elastic sphere of masses 2kg and 3kg moving in opposite directions with velocities $8ms^{-1}$ and $6ms^{-1}$ respectively collide with each other.

Find their velocities after the impact

$m_1 = 2kg$

$m_2 = 3kg$

$u_1 = 8ms^{-1}$

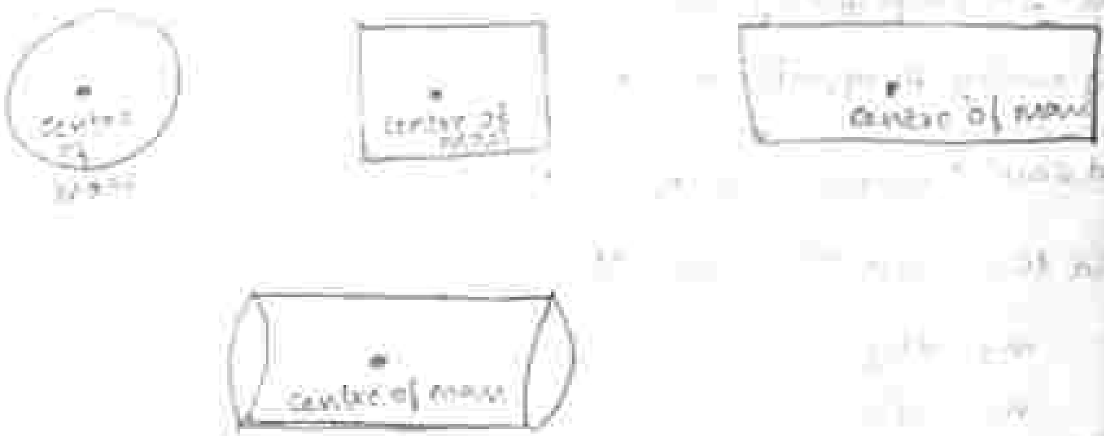
$u_2 = 6ms^{-1}$

According to the law of conservation of mom chapter

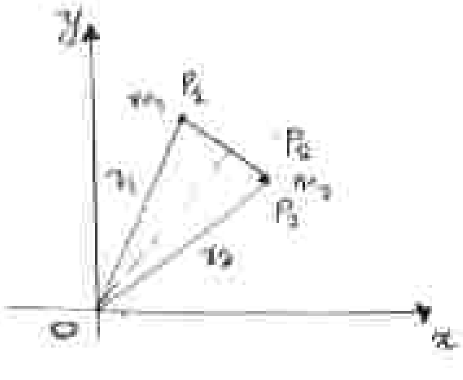
Systems of Particles and Rotational Motion

- Centre of mass

Where the whole mass of the body is supposed to be ~~cost~~ concentrated for describing its translatory motion



consider m_1 and m_2 are the two mass by total mass M the position vector P_1 & P_2 centre of mass of two particle system is R_{cm} of the particle is given by



$$R_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$(m_1 + m_2) R_{cm} = m_1 r_1 + m_2 r_2$$

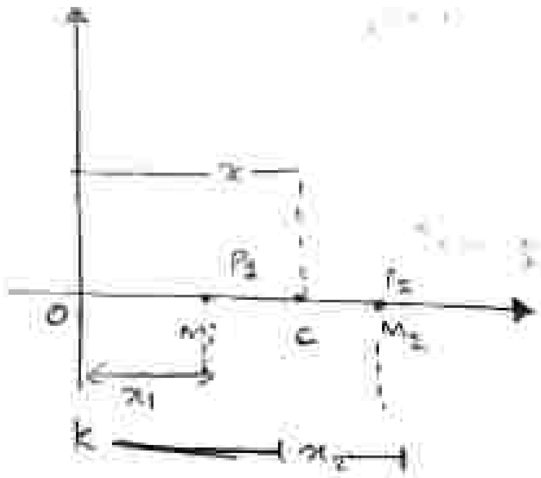
If $m_1 = m_2 = m$

$$(m+m) R_{cm} = m r_1 + m r_2$$

$$R_{cm} = \frac{\cancel{m}(r_1 + r_2)}{2\cancel{m}}$$

$$R_{cm} = \frac{(r_1 + r_2)}{2}$$

The line joining to the particle in x axis



$x_1 \rightarrow$ distance between "O" and P_1

$x_2 \rightarrow$ distance between "O" and P_2

$X \rightarrow$ distance between "O" and centre point

$$R_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$R_{cm} = \sum_{i=1}^{n} \frac{m_i x_i}{m_i}$$

$m_i = m_2 = m$

$$R_{cm} = \frac{m x_1 + m x_2}{m + m}$$

$$R_{cm} = \frac{\cancel{m}(x_1 + x_2)}{2\cancel{m}}$$

$$R_{cm} = X_{cm}$$

$$X_{cm} = \frac{x_1 + x_2}{2}$$

$$\Rightarrow$$

$$R_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum^{n} m_i}$$

$\sum m_i$ = Total mass or sum of all the masses

$$r_{cm} = x\hat{i} + y\hat{j} + z\hat{k}$$

For position vector in x, y, z axis

$$R_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

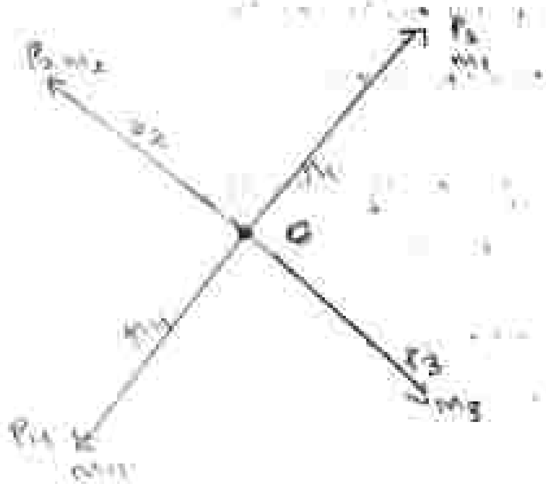
Centre of mass in y-axis

$$R_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

Centre of mass in Z axis

$$R_{cm} = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$$

The sum of mass moment of all particles of a system above its centre of mass is always equal to zero.



$r_1 = x\hat{i} + y\hat{j} + z\hat{k} \rightarrow$ for first position (m_1)

$r_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \rightarrow$ for 2^{nd} position (m_2)

$r_3 = x_3\hat{i} + y_3\hat{j} + z_3\hat{k} \rightarrow$ for 3^{rd} position (m_3)

$r_n = x_4\hat{i} + y_4\hat{j} + z_4\hat{k} \rightarrow$ for 4^{th} position (m_4)

$$R_{cm} = \frac{m_1r_1 + m_2r_2 + m_3r_3 + m_4r_4}{m_1 + m_2 + m_3 + m_4}$$

$$(m_1 + m_2 + m_3 + m_4) R_{cm} = m_1r_1 + m_2r_2 + m_3r_3 + m_4r_2$$

$$(m_1 + m_2 + m_3 + m_4) = m_1r_1 + m_2r_2 + m_3r_3 + m_4r_4$$

$$\underline{\underline{m_1r_1 + m_2r_2 + m_3r_3 + m_4r_4 = 0}}$$

$$R_{cm} = \frac{m_1r_1 + m_2r_2 + m_3r_3}{m_1 + m_2 + m_3}$$

$$r_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$R_{cm} = \frac{m_1(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + m_2(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) + m_3(x_3\hat{i} + y_3\hat{j} + z_3\hat{k})}{m_1 + m_2 + m_3}$$

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3}$$

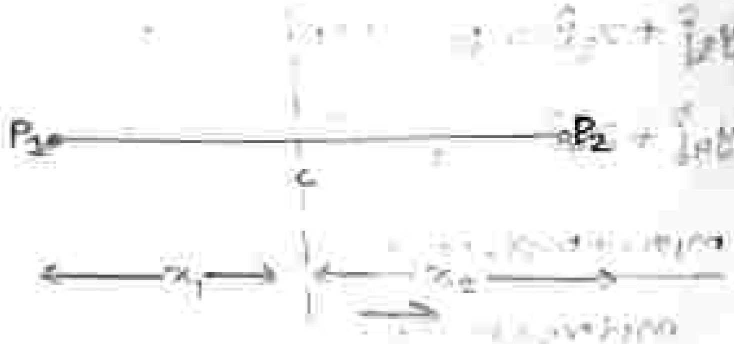
$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1+m_2+m_3}$$

$$z_{cm} = \frac{m_1z_1 + m_2z_2 + m_3z_3}{m_1+m_2+m_3}$$

Centre of mass for all coordinates

Centre of mass of two particles

Along x-axis



$$0 = \frac{m_1x_1 + m_2x_2}{m_1+m_2}$$

$$m_1x_1 + m_2x_2 = 0.$$

$(x_1 = -x_1)$ along x-axis

$$m_1(-x_1) + m_2x_2 = 0$$

$$-m_1x_1 + m_2x_2 = 0$$

$$m_2x_2 = m_1x_1$$

$$\frac{m_2}{m_1} = \frac{x_1}{x_2}$$

$$m_2 : m_1 = x_2 : x_1$$

2 : 1   2 : 1

Motion of centre of mass

(1) Position of centre of mass of the system

$$R_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$m_1 + m_2 = M$

$$R_{cm} = \frac{m_1 r_1 + m_2 r_2}{M}$$

$$\boxed{MR_{cm} = m_1 r_1 + m_2 r_2 + \ldots (m_n r_n}$$

This position of centre of mass

(2) Velocity of centre of mass

differ eq(1) w.r.t time

$$\frac{d(MR_{cm})}{dt} = \frac{d}{dt}(m_1 r_1) + \frac{d}{dt} m_2 r_2 + \text{---} \frac{d}{dt}(m_n r_n)$$

$$M\frac{dR_{cm}}{dt} = m_1 \frac{dr_1}{dt} + m_2 \frac{dr_2}{dt} + \ldots\text{--} m \frac{dr_n}{dt}$$

$$\boxed{MV_{cm} = m_1 v_1 + m_2 v_2 + \ldots\ m_n v_n \rightarrow 2}$$

This is a velocity of centre of mass of system

(3) acceleration of centre of mass

diff. eq^n (2) w.r.t time

$$\frac{d(MV_{cm})}{dt} = \frac{d}{dt}(m_1 v_1) + \frac{d(m_2 v_2)}{dt} \text{----} \frac{d}{dt}(m_n v_n)$$

$$M\frac{dv_{cm}}{dt} = m\frac{dv_1}{dt} + m_2\frac{dV_2}{dt} \ldots m\frac{dV_n}{dt}$$

$$\boxed{Ma_{cm} = m_1 a_1 + m_2 a_2 + \ldots m_n a_n}$$

This is acceleration of centre of mass

$$Wkt - F = ma$$

$$\boxed{F_{cm} = F_1 + F_2 + \ldots F_n}$$

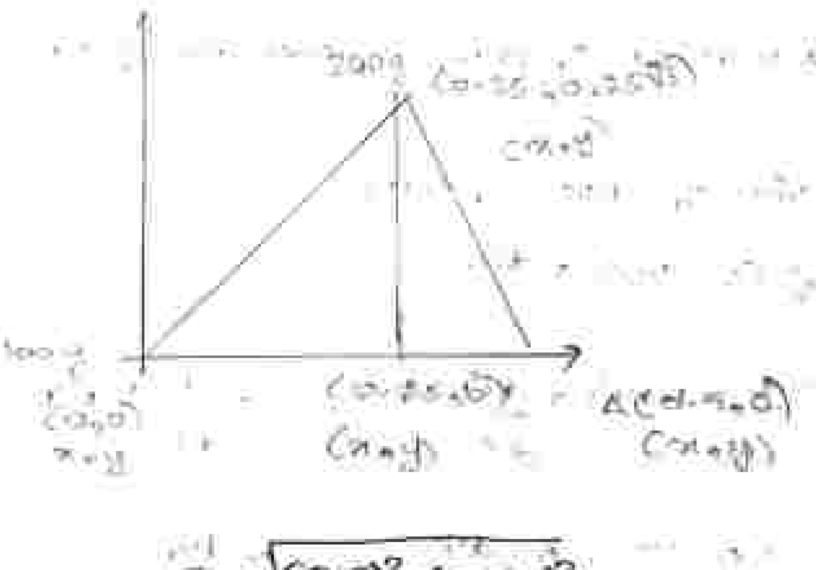
where ~~[illegible]~~ $F_{ext} = Ma_{cm}$

F_{ext} represents the sum of all external forces acting on the particle of the system

The centre of mass of the system of particles moves as if all the mass of the system was concentrated at the centre of mass and all the external forces were applied at that point.

Problem:

Find the centre of mass of 3 particles at the vertices of an equilateral triangle. The masses of the particle are 100g, 150g and 200g respectively. Each side of the equilateral triangle is 0.5m long



$$= \sqrt{(0.5)^2 + (0.25)^2}$$

$$BP = 0.25\sqrt{3}$$

Along x direction

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{100(0) + 150(0.5) + 120(0.25)}{100 + 150 + 200}$$

$$= \frac{75+50}{450}$$

$$= \frac{125}{450}$$

$$x_{cm} = \frac{5}{18} = 0.27$$

$$Y_{cm} = \frac{m_1Y_1 + m_2Y_2 + m_3Y_3}{m_1 + m_2 + m_3}$$

$$Y_{cm} = \frac{100(0) + 150(0) + 200(0.25\sqrt{3})}{450}$$

$$= \frac{0 + 0 + 86.60}{450}$$

$$= \frac{87}{450}$$

$$= 0.19$$

Centre of mass (x_{cm}, Y_{cm})

$(0.27, 0.19)$

Q.3. Find the centre of mass of uniform L shaped Lamina with dimensions as show. The mass of Lamina is 3kg.

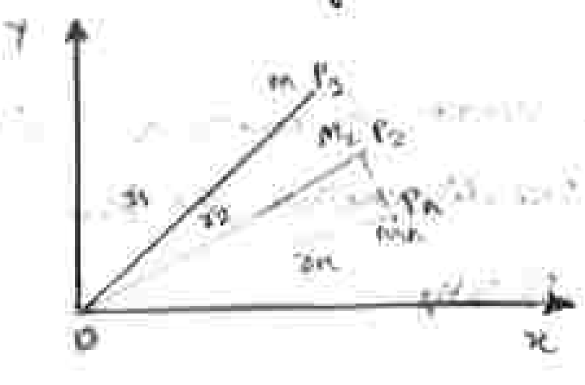
Ans: choosing x and Y axes as shown we have the coord of the vertices of the L-shaped lamina as given we can think of L shape to consist of 3 squares is 1kg. since the lamina is uniform. The centres of mass C_1, C_2 and C_3 of the squares are by symmetry their geometric centres and have coordinates (1/2), 1/2) (3/2, 1/2) (1/2, 3/2) respectively

Hence

$$X = \frac{1(1/2)+1(3/2)+1(1/2)\,kg\,m}{(1+1+1)kg} = \frac{5}{6}m$$

$$Y = \frac{(1(1/2)+1(1/2)+1(3/2)}{(1+1+1)\,kg} = \frac{5}{6}m.$$

centre of mass of n^{th} particle system



consider position of particle and mass of the particle and distance between the particle $r_1, r_2 \dots r_n$ and $m_1, m_2 \dots m_n$ and r_1, r_2, r_n the total mass of the system

$$\sum m_i = m_1 + m_2 + \dots m_n$$

$$M = \sum_{i=1}^{n} m_i$$

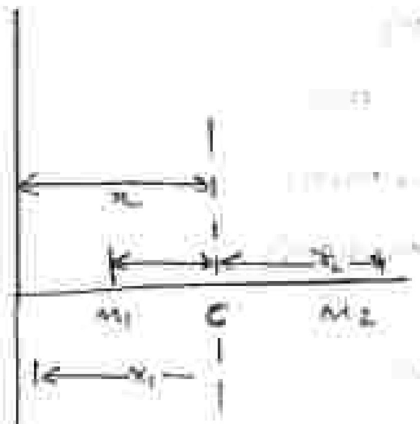
The sum of centre of mass of the system.

$$R_{cm} = \frac{m_1 r_1 + m_2 r_n \dots m_n r_n}{m_1 + m_2 + \dots m_n}$$

for n^{th}

$$R_{cm} = \frac{\sum_{i=1}^{n} m_i r_i}{M}$$

Centre of mass for one dimensional system



$r_1 = x - x_1$

$r_2 = x_2 - x$

$$m_1 r_1 + m_2 r_2 = 0 \rightarrow (1)$$

$$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$

$$m_1 r_1 = m_2 r_2$$

$$m_1 (x - x_1) = m_2 (x_2 - x)$$

$$m_1 x - m_1 x_1 = m_2 x_2 - m_2 x$$

$$m_1 x + m_1 x = m_1 x_1 + m_2 x_2$$

$$x(m_1 + m_2) = m_1 x_1 + m_2 x_2$$

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x = x_{cm}$$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Centre of mass of two particles can be regarded as mass waited mean of x_1 and x_2

Also, the distance between the particle

$$d = r_1 + r_2$$

$$r_1 = d - r_2$$

Replacing it in eq^n-(2)

$$m_1 r_1 = m_2 r_2$$

$$m_1(d - r_2) = m_2 r_2$$

$$m_1 d - m_1 r_2 = m_2 r_2$$

$$m_1 d = m_2 r_2 + m_1 r_2$$

$$m_1 d = r_2 (m_1 + m_2)$$

$$r_2 = \frac{m_1 d}{m_2 + m_1}$$

JEE mains.

$$d = r_1 + r_2$$

$$r_2 = d - r$$

From eq^n (3)

$$m_1 r_1 = m_2 r_2$$

$$m_1 r_1 = m_2 (d - r)$$

$$m_1 r_1 = m_2 d - m_2 r_1$$

$$m_1 r_1 + m_2 r = m_2 d$$

$(m_1+m_2)x_1 = m_2 d$

$x = \frac{m_2 d}{m_1+m_2}$

Application

$m_1 x_1 = m_2 x_2$

$\frac{m_1}{m_2} = \frac{x_2}{x_1}$

$x_1 = l - x$

$m_1 = m$

$m(l-x) = Mx$

$ml - mx = Mx$

$ml = Mx + mx$

$ml = x(M+m)$

$x = \frac{ml}{M+m}$ If θ it mentions $x = \frac{ml\cos\theta}{(M+m)}$

Consider

A widge of Mass (M) and base length (L) resting on a horizontal surface a principle momentum

M = mass of Balloon

m → mass of monkey

L → length of the balloon thread as per vertical direction y - axis

$$Y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$y_1 = L - \Delta y_2 \quad 0 = \frac{M(L - \Delta y_2) + m(\Delta y_2)}{M + m}$$

$$= \frac{ML - M\Delta y_2 + m\Delta y_2}{M + m}$$

$$0 = ML - \Delta y_2 (M + m)$$

$$\Delta y_2 (M + m) = ML$$

$$\Delta y_2 = \frac{ML}{M + m}$$

centre of mass of monkey ascends by Δy

centre of mass of balloon descends by Δy_2

Position of centre of mass will not change $\Delta y_{cm} = 0$

Vector product of two vectors

A vector product of two vectors A and B is a vector C

If 'a', 'b'

$c = a \times b$

$\vec{c} = \vec{a} \times \vec{b}$

$\vec{c} = ab \sin\theta \, \hat{n} \rightarrow$ cross product

Ex - $T = r \times f$

$T = rf \sin\theta$

c is ⊥ to the plane containing a and b.

$\vec{c} = \vec{a} \times \vec{b}$

$c = ab \sin\theta \, . \hat{n}$

$$a \times b \begin{vmatrix} \hat{i} & -\hat{j} & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Problems :

Find the scalar and vector product of two vectors

$a = 3\hat{i} - 4\hat{j} + 5\hat{k}$

$b = -2\hat{i} + \hat{j} - 3\hat{k}$

Scalar product

$$\begin{matrix} \hat{i} & -\hat{j} & k \\ 3 & -4 & 5 \\ -2 & 1 & -3 \end{matrix}$$

$a \cdot b = -6 - 4 - 15$

$= -10 + 15$

$a \cdot b = -25$

$$\left\{ \begin{matrix} \hat{i} \cdot \hat{i} = k1 \\ \hat{j} \cdot \hat{j} = 1 \\ k \cdot k = 1 \end{matrix} \right\}$$

Cross product

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & -4 & 5 \\ -2 & 1 & -3 \end{vmatrix}$$

$= 7\hat{i} - \hat{j} - 5k$

$$b \times a = \begin{vmatrix} \hat{i} & \hat{j} & k \\ -2 & 1 & -3 \\ 3 & -4 & 5 \end{vmatrix}$$

$= -7\hat{i} + \hat{j} + 5\hat{k}$

If vector $a = 2\hat{i} + \hat{j}$

If vector $b = \hat{i} - \hat{j} + 5\hat{k}$

Find both scalar product and vector product

$a \cdot b = 2 - 1 + 10$

$a \cdot b = 1$

Vector product

$$a \times b = \begin{vmatrix} \hat{i} & -\hat{j} & k \\ 2 & 1 & 0 \\ 1 & -1 & 5 \end{vmatrix}$$

$$\hat{i}\,[1 \times 5] - [0 \times -1]] - \hat{j}\,[2 \times 5 - 1 \times 0] + k\,[(2 \times -1) - (1 \times 1)]$$

$$= 5\hat{i} - 10\hat{j} - 3\hat{k}$$

$\vec{a} = 2\hat{i} + 3\hat{j} - k$

$\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$

$$a \times b \begin{vmatrix} \hat{i} & -\hat{j} & k \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix}$$

$\hat{i}\,(6 - 1) - j\,(4 - (-1)) + k\,(-2 - 3)$

$a \times b = 5\hat{i} - 5\hat{j} - 5k$

$b \times a = 5\hat{i} + 5\hat{j} + 5\hat{k}$

Dot product

$a \cdot b = 2 - 3 - 2$

$2 - 5$

$= -3$

Torque

Torque is defined as force × ⊥ distance and also known as movement of force

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|}$$

$\tau = F \times \perp^{r}$ distance

$\tau = r \times F \sin\theta \, \hat{n}$

$r \times \perp^{r}$ distance

$F \rightarrow$ Force

If $\theta = 0°$

$\tau = r \times F \sin\theta$

$r \times F(0)$

$\tau_{min} = 0$

when $\theta = 90°$

$\tau = r \times F \sin 90°$

$r \times F(1)$

$\tau_{max} = r \times F$

Angular velocity and angular acceleration

Angular displacement

The movement or the displacement of the particle in its axis

Denoted as (θ)

Angular momentum [L]

The movement of the linear movement of the particles about that axis

Angular momentum = ⊥ distance × linear momentum
[axis of rotation

$$L = r \times \vec{P}$$

consider a particle (P) of mass (m) rotating about an axises through origin in x, y plane. suppose a particle has linear momentum which makes angle (θ) with

The position vector (r)

If $\theta = 0°$ or $180°$

$rp \sin(0)°$

$L = rp(0)$

$L_{min} = 0$

$L = rp\sin\theta$

If $\theta = 90°$

$L = rp\sin(90°)$

$rp\sin(1)$

$L_{max} = rp$

Angular velocity

The rate of change of angular momentum with respect to time is called angular velocity

$W = vr$

$v = \frac{W}{r}$

$W = \frac{dl}{dt}$

$W = \frac{d(vr)}{dt}$

S.I unit : $rad s^{-1}$

Note:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 - u^2 = 2as$

Angular equations

$W = W_0 + \alpha t$

$\theta = w_0 t + \frac{1}{2}\alpha t^2$

$w^2 - w_0^2 = 2\alpha\theta$.

Relation between torque and angular momentum

Torque $= \vec{T} = r \times f$

Angular momentum $= \vec{L} = \vec{r} \times \vec{p}$

From eqⁿ (1)

$\vec{L} = \vec{r} \times \vec{p}$

or $\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$

$\vec{r} = \frac{d\vec{p}}{dt} + \vec{p} \times \frac{d\vec{r}}{dt}$

$= \vec{r} \times \vec{F} + m\vec{v} \times \vec{v}$

$\frac{dv}{dt} = r \times \vec{F} + 0 \quad \frac{dL}{dt} = \vec{T}$

Torque and angular momentum

$L_1, L_2, L_3 \ldots L_n$ Angular momentum of particles

$r_1, r_2, r_3 \ldots r_n$ is $\perp^r$ distance in axis rotation

$P_1, P_2, P_3 \ldots P_n$ is linear momentum

$$L = \sum_{i=1}^{n} r_i \times p_i$$

In the form torque

$$T = \frac{dL}{dt}$$

$T = T_1 + T_2 + T_3 + \ldots T_n$

$T_{ext} = r_1 \times F_1 \times r_2 \times F_2 \times r_3 \times F_3 + r_n \times F_n$

$$T_{ext} = \sum_{i=1}^{n} r_i \times F_i$$

$$T_{ext} = \frac{dL}{dt}$$

Rigid Body

An ideal object force shape and size can never be change

A rigid Body is said to be in mechanical equilibrium

It has both translational & rotational equilibrium

$F_1 + F_2 + F_3 \ldots F_n = F_{ext}$

Principle of momentum

Two forces F_1 & F_2 parallel to each other and usually ⊥ to the leaver and act on the lever at distance d_1 & d_2 respectively

Rotation motion

$$R = F_1 + F_2$$

Based LoM $m_1 r_1 = m_2 r_2$

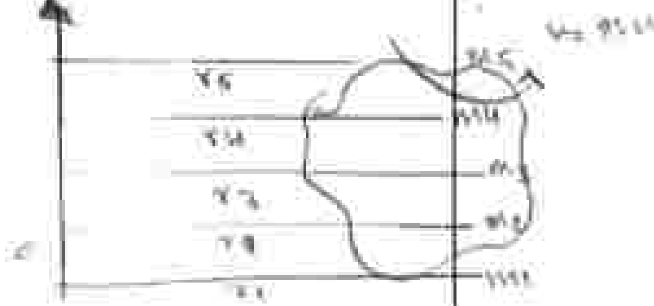
$$F_1 d_1 = F_2 d_2$$

$$\frac{F_1}{F_2} = \frac{d_2}{d_1} \qquad d_2 > d_1$$

Moment of Inertia

The moment of inertia of a rigid body about a fixed axis is defined as the sum of the product of the masses of the particles consisting the body and the squares of their respective distances from the axis of rotation.

Consider a rigid body rotating with uniform angular velocity (ω) about a vertical axis through origin O, the body consist of n particle of masses $m_1, m_2, m_3 \ldots m_n$ and distances $R_1, R_2, R_3 \ldots R_n$ respectively from the axis of rotation. The moment of inertia of the body about the axis is



$$MI = MI_1 + MI_2 + MI_3 \dots MI_n$$

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \dots m_n r_n^2$$

$$I = \sum_{i=1}^{n} m_i r_i^2$$

$$I = \sum_{i=1}^{n} m_i r_i^2$$

SI unit of moment of inertia is kgm^2

Dimensional formula $[M^1 L^2 T^0]$

It depends on mass of the body size and shape distribution of mass about the axis rotation

Relation between Rotational Kinetic energy and moment of inertia

$KE = \frac{1}{2}mv^2$ Consider a rigid body of rotating about an axis with uniform angular velocity ω. The body may consists n particles of masses $m_1, m_2, m_3 \dots m_n$ and distances $R_1, R_2, R_3 \dots R_n$ from the axis of rotation the angular velocity ω of all the n particles is same.

$$KE = \frac{1}{2}mv^2$$

when an object in linear motion

$$V = r\omega$$

for $V = r_1\omega_1$, $V = r_2\omega_2$, $V = r_3\omega_3$, $V = r_n\omega_n$

for sum of K.E

$$K.E = KE_1 + KE_2 + KE_3 \dots KE_n$$

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 \ldots \frac{1}{2}m_nv_n^2$$

Replace $v = r\omega$

$$K.E = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \frac{1}{2}m_3r_3^2\omega^2 \ldots \frac{1}{2}m_nr_n^2\omega^2$$

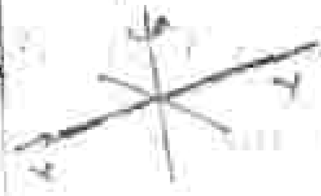
$$I = m_1r_1^2$$

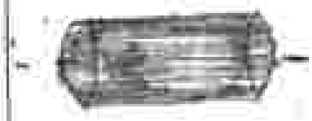
$$K.E = \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2 + \frac{1}{2}I_3\omega^2 \ldots \frac{1}{2}I_n\omega^2$$

$$KE = \frac{1}{2}I$$

$$I = 2KE$$

Moments of inestia

Z	Body	Axis	Figure	I
1.	Thin circular ring radius R	Perpendicular to plane at centre		MR^2
2.	Thin circular ring radius R	Diameter		$\frac{MR^2}{2}$
3.	Thin rod length l	Perpendicular to rod at mid point		$\frac{ML^2}{12}$
4.	Circular disc radius R	Perpendicular to disc at centre		$\frac{MR^2}{2}$
5.	Circular disc radius R	Diameter		$\frac{MR^2}{4}$

6.	Hallow cylinder radius R	Axis of cylinder		MR^2
7.	Solid cylinder R	Axis of cylinder		$\frac{MR^2}{2}$
8.	Solid sphere R	Diameter		$\frac{2MR^2}{5}$

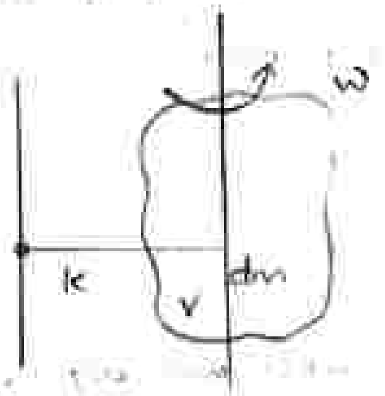
A ~~circular motion~~ Radius of ~~direction~~ gyration

$I = Mr^2$ The radius of gyration of rigid body

$I = mK^2$ position of axis of rotation distribution

$K^2 = \frac{I}{m}$ of mass about the axis of rotation

$K = \sqrt{\frac{I}{m}}$ It is the effective distance of all the particles of a body rotating about the given axis



$I = mr^2$

$I = mK^2$

$K^2 = \frac{I}{m}$

$K = \sqrt{\frac{I}{m}}$

$$k = k_1 + k_2 + k_3 \ldots k_n$$

$$k = \sqrt{\frac{I_1}{M}} + \sqrt{\frac{I_2}{M}} + \sqrt{\frac{I_3}{M_3}} \ldots \sqrt{\frac{I_n}{M_n}}$$

$$k = \sum_{i=1}^{n} \sqrt{\frac{I_n}{M_n}}$$

For system of particles

$$\text{For KCET} \sqrt{\frac{I_n}{M_n}} = \sqrt{\frac{M_1 r_1^2}{M_1}} + \sqrt{\frac{m_2 r_2^2}{m_2}} + \ldots \sqrt{\frac{m_n r_n^2}{m_n}}$$

$$= \sqrt{\frac{m_1 r_1^2 + m_2 r_2^2 + \ldots m_n r_n^2}{m_1 + m_2 + m_3 \ldots m_n}}$$

$$= \sqrt{\frac{M_1(r_1^2 + r_2^2 + r_3^2 \ldots r_n^2)}{n M_{net}}}$$

$$k = \sqrt{\frac{r_1^2 + r_2^2 + \ldots r_n^2}{n}}$$

Angular

when a torque acts on a body capable of rotation about an axis it produces an angular acceleration in the body

$$\alpha = \frac{Dw}{Dt}$$

The linear acceleration will depend on their distances $r_1, r_2, r_3 \ldots r_n$ from its axis of rotation

Consider a particle P of mass m at a distance r_1 from the axis of rotation.

Let, its linear velocity V

$a = r\alpha$

Linear acceleration = Angular acceration

$\tau = F \times r$

$F = ma$

$F = mr\alpha$

$\tau_1 = m_1 r_1 \alpha \times r$

$\tau_1 = m_1 r^2 \alpha_1$

$\tau_2 = m_2 r_2^2 \alpha_2$

$\tau_3 = m_3 r_3^2 \alpha_3$

Torque of the n^{th} particle

$\tau = \tau_1 + \tau_2 \dots + \tau_n$

$\tau = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + \dots m_n r_n^2 \alpha$

$(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots m_n r_n^2)$

$\tau = (I + I_2 + I_3 \dots I_n)$

$\tau = I\alpha$

Relation between angular momentum and moment of inertia

Consider $m_1, m_2, m_3 \dots + m_n$.

For masses of particle r_1, r_2, r_3 r_n are distance

linear velocity $v = r\omega$

$v_1 = r_1 \omega$

$v_2 = r_2 \omega$

$v_3 = r_3 \omega$

$P = mv$

$P = mr\omega$

$L = Pr$

$L = mr\omega r$

$L = mr^2 \omega$

$L_1 = m_1 r_1^2 \omega$

$L_2 = m_2 r_2^2 \omega$

$L_3 = m_3 r_3^2 \omega$

$L = L_1 + L_2 + L_3 + \dots L_n$

$L = m_1 r_1^2 \omega + m_2 r_2^2 \omega^2 + m_3 r_3^2 \omega + \dots m_n r_n^2 \omega$

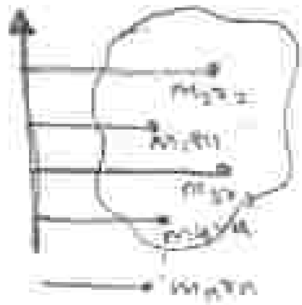
$L = (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots m_n r_n^2)\omega$

$L = (I + I_2 + I_3 \dots I_n)\omega$

$L = I\omega$

Centre of gravity

Consider a particle of masses $m_1, m_2, m_3 \dots m_n$ located at respective perpendicular distance of $r_1, r_2, r_3 \dots r_n$ the moment due to weights of individual particle



$CG = m_1 r_1 g + m_2 r_2 g + m_3 r_3 g \dots m_n r_n g$

$CG = (m_1 r_1 + m_2 r_2 + \dots m_n r_n) g$

g = acc-on due to gravity

$CG = \sum_{i=1}^{n} m_i r_i$

Linear Motion	Rational Motion about a fixed axis
Displacement x	Angular displacement θ
Velocity $v = \frac{dx}{dt}$	Angular velocity $\omega = \frac{d\theta}{dt}$
Acceleration $= \frac{dv}{dt}$	Angular acceleration $\alpha = \frac{d\omega}{dt}$
Mass M	Moment of Inertia I
Force $F = Ma$	Torque $T = I\alpha$
Work $dW = Fds$	Work $W = Td\theta$
Kinetic energy $K = \frac{mv^2}{2}$	Kinetic energy $K = \frac{I\omega^2}{2}$
Power $P = Fv$	Power $P = T\omega$
Linear momentum $P = Mv$	Angular momentum $L = I\omega$

Example 6.20: A fly wheel of moment of inertia 100 kg m^2 and radius 1 m is accelerated by applying a tangential force of magnitude 20N. Find its angular velocity 5 s after the start

Torque $(T) = (20N)(1m) = 20N\text{-}m$

Angular acceleration

$$= \frac{T}{I} = \frac{20N}{100kg\text{-}m^2} = 0.2\ rad/s^2$$

As "α" is constant

$$\omega = \omega_0 + \alpha t = 1\ rad/s$$

Example : A ballet dancer spins about a vertical axis & at 120 rpm with arms outstretched, with her arms folded the moment of inertia about the axis of rotation decreases by 40%. calculate the new rate of rotation

$$I_1\omega_1 = I_2\omega_2$$

$$I_1 2\pi n_1 = I_2 2\pi n_2 = I_1 n_1 = I_2 n_2$$

$$\frac{n_2}{n_1} = \frac{I}{I_2}$$

$$\frac{40I}{100} = \frac{60I}{100} = \frac{6I}{10}$$

$$\therefore \frac{n_2}{n_1} = \frac{I}{\left(\frac{6I}{10}\right)} = \frac{10}{6}$$

$$\frac{n_2}{n_1} = \frac{10}{6}$$

$$\therefore \frac{10}{6} \times 120 = 10 \times 20 = 200\ rpm$$

Example 6.15 A uniform bar of length 6a and mass 8m lies on a smooth horizontal table. Two point masses m and 2m moving in the same horizontal plane with speeds 2v and v respectively, strike the bar and stick to the bar after collision.

Calculate

(a) Velocity of centre of mass

(b) Angular velocity

(c) Total kinetic energy

$-2m \times v + m \times 2v + 0 = (2m + m + 8m) \times V$

$V = 0$

(b) As $\tau_{ext} = 0$ Angular momentum of the system is conserved

i.e. $m_1 v_1 r_1 + m_2 v_2 r_2 = (I_1 + I_2 + I_3)\,\omega$

$2mva + m(2v)(2a) = \left(2m(a)^2 + m(2a)^2 + 8m \times \left(\frac{6a^2}{12}\right)\right)\omega$

$6mva = 30ma^2\omega$

$\omega = \frac{v}{5a}$

(c) As from part (a) and (b) it is clear that, the system has no translatory motion but only rotatory motion

$E = \frac{1}{2} I \omega^2 = \frac{1}{2}(30ma^2)\left[\frac{v}{5a}\right]^2 = \frac{3}{5}mv^2$

8-09-23

Chapter

Gravitation

Gravitation is the force of attraction between any two bodies in the universe

Gravity :-

Gravity is the force of attraction between earth and any object lying on near its surface.

Free fall :-

Motion of the body under the influence of gravity alone is called a free fall

Acceleration due to gravity :-

The Acceleration produced in a freely falling body under the influence of gravitational pull of the earth is called acceleration due to gravity

Weight of the Body

The Gravitational force with which a body is attracted towards the centre of the earth

$w = M \times g$

SI unit :- ~~kgm~~ $kgms^{-2}$ or Nm^{-2}

It is a vector quanity

* Newton universal law of gravitation

Any two particles

Newtons universal law: Any two particles in the universe attract each other with a force is directly proportional to the magnitude of product of their masses and inversely proportional to the square of the distance between them and direction of the force is along the line joining two particles.

If M_1, M_2 are two masses of the particle and d is the distance between them, the gravitational force is

$$F \propto \frac{m_1 m_2}{d^2}$$

$$F \propto \frac{(m_1 m_2)}{d^2}$$

$$\boxed{F = G\frac{m_1 \times m_2}{d^2}}$$

The value of G is $6.67 \times 10^{-11}\ Nm^2kg^{-2}$

$$F = \frac{Gm_1m_2}{d^2}$$

$$G = \frac{Fd^2}{m_1m_2} = \frac{kgms^{-2} \times m^2}{kg\,kg} \Rightarrow \frac{Nm^2}{kg \times kg} = Nm^2kg^{-2}$$

Dimensional formula = $M^{-1}L^3T^{-2}$

Vector form of universal law of Gravitation

d = r

$$F_{12} = \frac{G|M_1 M_2|}{\vec{r}^{\,2}}\,\hat{r}$$

$$\vec{F}_{12} = \frac{G|M_1 \times M_2|}{\vec{r}_{12}^{\,3}}\,\vec{r}$$

$$\vec{F}_{21} = \frac{G|M_1 M_2|}{\vec{r}^{\,2}}\,\hat{r}$$

$$F_{21} = \frac{G|M_1 \times M_2|}{\vec{r}^{\,3}}\,\vec{r}$$

$$F_{12} = -F_{21}$$

$$F_{12} + F_{21} = 0 \Rightarrow \text{Net force}$$

$$F_{12} = -F_{21} = \frac{G|M_1 \times M_2|}{\vec{r}^{\,3}}\,\vec{r}$$

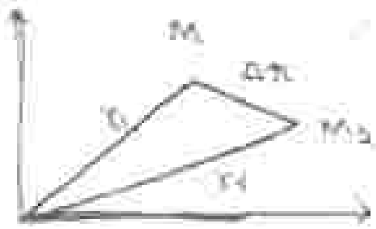
r_{12} with to 2

r_{12} = unit vector from A to B

r_{21} = unit vector from B to A

F_{12} = Gravitation force exerted on both A by body B

F_{21} = Gravitation force exerted by body B by Body A



$$= \frac{G|M_1 \times M_2|}{(r_2 - r_1)^3}\,(\vec{r}_2 - \vec{r}_1)$$

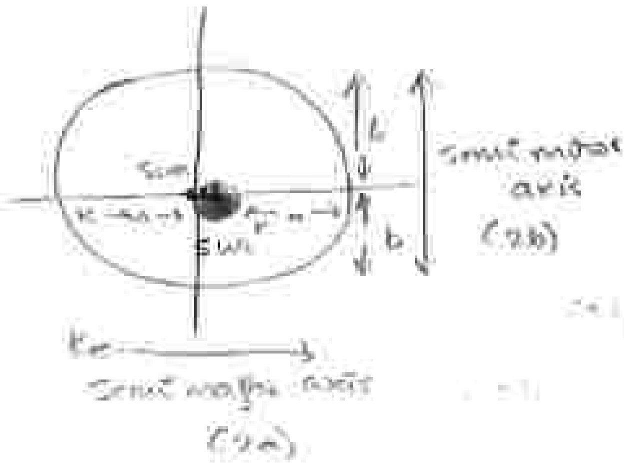
Characteristics of gravitational force

- It is always attractive force
- It is independent of medium
- It is conservative force
- It is central force

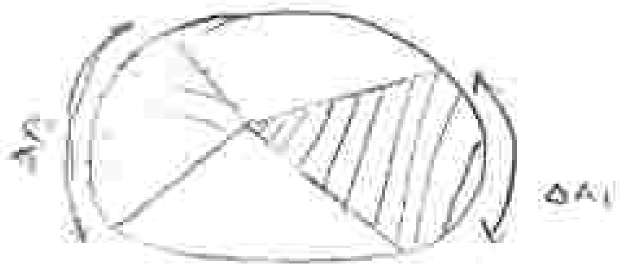
Kepler's law

First law : law of orbits

All planets move in a elliptical orbits with ~~sun~~ the ~~a~~ sun at one of the ~~Foci~~ Focii of the elipse.

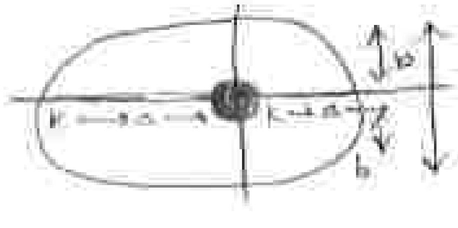


(2) law of area



The line joining the planet to the sun sweeps equal areas in equal interval of time.

(3) law of periods.



The square of the time period of the revolution of a planet is proportional to the cube of the semi ~~maxi~~ major axis of the elliptical part traced by the planet

$$T^2 = a^3$$

Deduction of newton's law of gravitation from Kepler's law

$$F_c = mr\omega^2$$

$$= mr\left(\frac{2\pi}{T}\right)^2$$

$$= mr\left(\frac{4\pi}{T^2}\right)$$

$$= mr\frac{4\pi^2}{T^2} \rightarrow (1)$$

From Kepler's 3rd law

$$T^2 \propto a^3$$

$$T^2 = ka^3 \qquad a = r$$

$$T^2 = kr^3$$

$$= \frac{mr\,4\pi^2}{kr^3}$$

$$F_c = \frac{m}{k} \cdot \frac{4\pi^2}{r^2}$$

$$F_c = \frac{m}{r^2}\left(\frac{4\pi^2}{k}\right) \rightarrow (2)$$

$$= \frac{4\pi^2}{k} \propto M$$

$$= \frac{4\pi^2}{k} = GM \rightarrow (3)$$

Substitute eq(3) in (2)

$$F_c = \frac{m}{r^2} \cdot GM$$

$$F_c = \frac{GmM}{r^2}$$

$\frac{4\pi^2}{k}$ - It is directly proportional to the mass of the planet it should alom also be directly proportional to the mass of the sun

* Acceleration due to the gravity of the Earth

From universal law of gravitation

$$F = \frac{Gm_1m_2}{d^2}$$

$m_1 = M$

$m_2 = m$

$$F = \frac{GMm}{d^2} \rightarrow (1)$$

From newton's second law

$F = ma$

$a = g$

$$F = mg \rightarrow (2)$$

Comparing eq^n (1) and (2)

$$mg = \frac{GMm}{d^2}$$

$$g = \frac{GM}{d^2}$$

$G = 6.67 \times 10^{-11} \, Nm^2kg^{-2}$

$M_e = 5.97 \times 10^{24} \, kg$

$d = 6.4 \times 10^6 m$

$d = 6400 km$

$$g = \frac{GM}{d^2}$$

$$\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.4 \times 10^6)^2}$$

$= \frac{39.81 \times 10^{3}}{40.96 \times 10^{12}}$

$= 0.971 \times 10^{12} \times 10^{-12}$

$= 0.97 \times 10$

$g = 9.7 ms^{-2}$

$g = 9.7 \simeq 10 ms^{-2}$

Imagine a light planet revolving around a very massive star in a circular orbit of radius r with period of revolution T on what power of r will the square of the time period depends if the gravitational force of attraction between the planet and the star is proportional to $r^{-5/2}$

"T" relation "r"

$F_c \propto r^{-5/2}$

$\frac{mv^2}{r} = kr^{-5/2}$

$\frac{mv^2}{r} = \frac{k}{r^{5/2}}$

$v^2 = \frac{kr}{r^{5/2} \times m}$

$v = \sqrt{\frac{kr}{r^{5/2} \times m}}$

$T = \frac{2\pi}{\omega}$

$T = \frac{2\pi r}{v}$

$T = 2\pi r \sqrt{\frac{kr}{r^{5/2} \times m}}$

$T \propto r \sqrt{\frac{r}{r^{5/2}}}$

$T \propto r \sqrt{r^{-5/2} \cdot r^{1}}$

$T \propto r \sqrt{r^{-3/2}}$

$-\frac{5}{2}-1$

$-\frac{5+2}{2}$

$=-\frac{3}{2}$

$T \propto r^{-\frac{3}{2} \times \frac{1}{2}}$

$T \propto r^1 \cdot r^{-3/4}$ X

$T \propto r^{1/4}$

$V = \sqrt{\frac{K}{r^{5/2-1}}}$

$\sqrt{\frac{K}{r^{3/2} m}} \rightarrow (2)$

Replace eq(2) by (1)

$T = \frac{2\pi r}{\sqrt{\frac{K}{r^{3/2} m}}}$

$T = 2\pi r \times \sqrt{\frac{r^{3/2} m}{K}}$

$T = r\sqrt{r^{3/2}}$

$= r \cdot r^{3/4}$

$T = r^{7/4}$

$T = r^{7/2 \times \frac{1}{2}}$

$T^2 = r^{7/2}$

A saturn year is 29.5 times the earth the earth year how far is the saturn from the sun if the earth is 1.50×10^8 km away from the sun

$T_S^2 \propto R_S^3$

$T_E^2 \propto R_E^3$

$T_s = 29.5\, T_E$

$R_E = 1.50 \times 10^8$

$$\left(\frac{T_s}{T_E}\right)^2 = \left(\frac{R_s}{R_E}\right)^3$$

$$\left(\frac{29.5\, T_E}{T_E}\right)^2 = \left(\frac{R_s}{1.50 \times 10^8}\right)^3$$

$$(29.5)^2 \times (1.50 \times 10^8)^3 = R_s^3$$

$$870.25 \times 3.37 \times 10^{24} = R_s^3$$

$$(2932.7)^{1/3} \times 10^8 = R_s$$

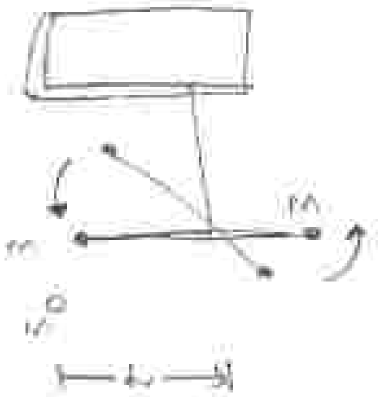
$$54.15$$

$$9.54 \times 1.50 \times 10^8 = R_s$$

$$14.31 \times 10^8 = R_s$$

$$1.4 \times 10^9\, m = R_s$$

The Gravitational constant



$$F = \frac{GMm}{d^2}$$

Torque $= F \times L$

Γ = couple per unit twist

$K\theta = \Gamma$

$$F = \frac{GMm}{d^2}$$

$$\frac{T}{L} = \frac{GMm}{d^2}$$

$$T = \frac{GMm}{d^2} \cdot L$$

$$k\theta = \frac{GMm}{d^2} \cdot L$$

$$\theta = \frac{GMm}{d^2} \cdot \frac{L}{k}$$

The mean ~~diff~~ distance of mass from the sun is 1.524 times that of earth from the sun find the number of years required for mass to ~~take~~ make 1 revolution about the sun

Given: Distance ~~from~~ of mars sun to earth = 1.524 times of earth

To find

1 revolution about the sun

$$T^2 \propto R^3$$

$$\left(\frac{T_s}{T_e}\right)^2 \propto \left(\frac{R_s}{R_E}\right)^3$$

$$R_m = 1.524 R_E = R_s$$

$$\left(\frac{T_s}{T_e}\right)^2 = \left(\frac{1.524 R_E}{R_E}\right)^3$$

$$T_S^2 = T_E^2 (1.524)^3$$

$$T_e^2 = 1^2 (1.524)^3$$

$$T_s^2 = (1.524)^3$$

$$T_s = 1.88$$

A star 2.5 times the mass of the sun and collapse to a size of 12km rotates with a speed of 1.5 revolution per second. Will an object placed on its equator remain struck to itself is due to gravity

Mass = 2×10^{30}kg

$$g = \frac{GM}{R^2}$$

$$g = \frac{6.67\times10^{-11}\times2\times10^{30}}{(1.4)^2} \quad \frac{6.67\times10^{-11}\times2.5\times2\times10^{30}}{(12000)^2}$$

$$= \frac{3.34\times2\times10^{30-11}}{2.25} \quad = \frac{13.34\times10^{30-11}}{144000}$$

$$= \frac{13.34\times10^{19}}{(72000)} \quad = 2.3\times10^{12}ms^{-2}$$

centrifugal acceleration $= r\omega^2$

$$= r(2\pi\nu)^2$$

$$= 12000(2\pi\times1.5)$$

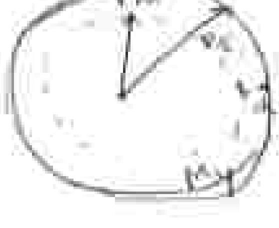
$$= 12000\left(2\times\frac{22}{7}\times1.5\right)$$

$$= 12000\left(\frac{44}{7}\times1.5\right)$$

$$= 12000\left(\frac{660}{7}\right)$$

$$= 1.1\times10^6 ms^{-2}$$

Relation between universal gravitation constant g and acceleration due to gravity



Consider a point mass m & r is the distance from the centre,

capit M_R = Mass of the assumed sphere

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{GmM_r}{r^2}$$

M_E = Mass of the Earth

R_E = Radius of earth

$$\frac{m}{V} = \frac{M_E}{\frac{4}{3}\pi R_E^3}$$

ρ = density of the Earth sphere

$$M_r = \frac{4}{3}\pi (R_E)^3 \times \rho$$

Replace eq^n (2) in (3)

$$M_r = \cancel{\frac{4}{3}}\pi R_E^3 \times \frac{M_E}{\cancel{\frac{4}{3}}\pi R_E^3}$$

$$M_r = \frac{M_E r^3}{R_E^3}$$

Replace M_r in eq^n (1)

$$\boxed{R_E^2 g_E = GM_E}$$

g_E = gravity due to acc^{en}

R_E = Radius

M_E = Mass

G = Gravity

$$F = \frac{GmM_r}{r^2 \times R_E^3}$$

$$= \frac{GmM_E r}{R_E^3}$$

$$= \frac{GmM_E R_E}{R_E^3} = \frac{GmM_E}{R_E^2}$$

$$mg_E = \frac{GmM_E}{R_E^2}$$

$$g_E = \frac{GM_E}{R_E^2}$$

Acceleration due to gravity above the earth surface [Altitude]

5 marks

$h \rightarrow$ Height of the point mass

$m \rightarrow$ point mass

$M \rightarrow$ Mass of the earth

$R \rightarrow$ Radius of the earth

$$F = \frac{GmM}{r^2}$$

$$F = \frac{GmM}{(h+R)^2}$$

$$mg = \frac{GmM}{(h+R)^2}$$

$$g = \frac{GM}{(h+R)^2}$$

$$g = \frac{gR^2}{(h+R)^2}$$

$$g = g_n\left(\frac{R}{h+R}\right)^2$$

$$g = g_n\left(\frac{R+h}{R}\right)^{-2}$$

$$g_n\left(1+\frac{h}{R}\right)^{-2}$$

Applying the Binomial theorem

$$\{(1+x)^n = 1+nx\}$$

$$\left(1+\frac{h}{R}\right)^{-2} = 1+(-2)\frac{h}{R}$$

$$g = g_n\left(1-\frac{2h}{R}\right)$$

consider earth to be homogonous uniform sphere of radius capital R, density $\rho \rightarrow$ density

$$g = \frac{GM}{R^2}$$

$M_s \rightarrow$ Mass of the earth in a solid sphere

$m \rightarrow$ point mass

$R \rightarrow$ Radius of earth

$(R-d) \rightarrow$ distance between centre to point mass

$$\frac{M_s}{M} = \frac{\cancel{\frac{4}{3}\pi}(R-d)^3}{\cancel{\frac{4}{3}\pi}(R)^3}$$

$$\boxed{\rho_s = \frac{M_s}{V_s} = \frac{M_s}{\frac{4}{3}\pi(R-d)^3} \qquad \rho = \frac{m}{v} = \frac{M}{\frac{4}{3}\pi(R)^3}}$$

$$\frac{M_s}{M} = \frac{(R-d)^3}{R^3}$$

$$F = \frac{GMm(R-d)^3}{R^3(R-d)^2}$$

$$F = \frac{GMm(R-d)}{R^3}$$

$$\cancel{m}g = \frac{GM\cancel{m}(R-d)}{R^3}$$

$$g = \frac{GM(R-d)}{R^3}$$

$$g = \frac{g_d R^2(R-d)}{R^3}$$

$$g = g_d\frac{(R-d)}{R}$$

$$g = g_d\left(1 - \frac{d}{R}\right)$$

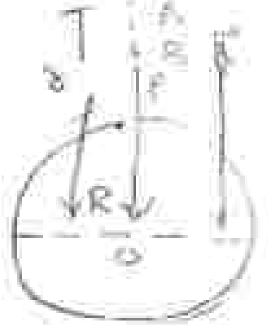
Note: g decreases as depth d increases.

• g becomes zero at the centre of earth

Gravitational potential energy

Gravitational potential energy is the energy stored in an object due to its position above the earth surface

Consider an point mass m and mass of the Earth M the distance between point mass and centre mass of the earth is r. The ~~force between~~ force between the point mass and centre of the mass is.



Mass of the earth
M → Point mass

W = E
W = ΔKE

$$F = \frac{GMm}{r^2}$$

The object from ∞

$$F = \frac{GMm}{x^2}$$

$$dw = F \cdot dx$$

$$\int_0^w dw = \int_\infty^r F \cdot dx$$

$$[w]_0^w = \int_\infty^r \frac{GMm}{x^2} \cdot dx$$

$$w - 0 = GMm \int_\infty^r \frac{1}{x^2} dx$$

$$w = GMm \int_\infty^r x^{-2} dx$$

$$w = GMm \left[\frac{x^{-2+1}}{-2+1} \right]_\infty^r$$

$$GMm \left[\frac{x^{-1}}{-1} \right]_\infty^r$$

$$GMm \left[-\frac{1}{x} \right]_\infty^r$$

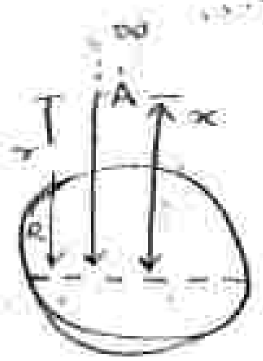
$$w = GMm \left[-\frac{1}{r} - \cancel{\frac{1}{\infty}} \right]$$

$$GMm \left[-\frac{1}{r} \right]$$

$$w = -\frac{GMm}{r}$$

$$U = -\frac{GMm}{r}$$

*** Escape speed



from universal law of gravitation

$$F = \frac{GMm}{r^2}$$

$$F = \frac{GMm}{x^2}$$

Force from R to ∞

$W = F.x$

$dW = F.dx$

Integrate on both sides

$\int dW = \int F.dx$

$$\int_0^W dW = \int_R^\infty F.dx$$

$$[W]_0^W = \int_R^\infty \frac{GMm}{x^2}\,dx$$

$$= GMm \int_R^\infty x^{-2}\,dx$$

$$= GMm \left[-\frac{1}{x}\right]_R^\infty$$

$$= GMm \left[\frac{-1}{\infty} - \left[-\frac{1}{R}\right]\right]$$

$$= GMm \left[\frac{1}{R}\right]$$

$$W = \frac{GMm}{R}$$

Change in K.E $= \frac{1}{2}mv^2$

$v = v_e$

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

$$V_e^2 = \frac{2GM}{R}$$

$$V_e^2 = \frac{2gR^2}{R} \quad \{GM = gR^2\}$$

$$V_e^2 = 2gR$$

$$V_e = \sqrt{2gR}$$

$g = 9.8 \quad R = 6400\,km$

$$V_e = \sqrt{9.8 \times 6400 \times 2 \times 10^{3}}$$

$$V_e = \sqrt{1.2544 \times 10^{4}}$$

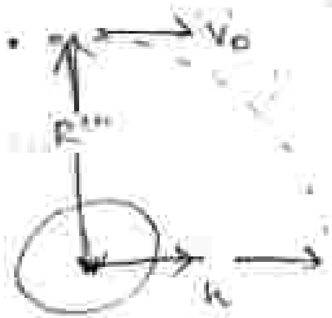
$$= 112 \times 10^2\, m/s$$

$$= 11.2 \times 10^3\, m/s$$

$$= 11.2\, km/s$$

Orbital velocity

~~Orbital velocity~~ ~~is~~ Orbital velocity is the velocity required to put the satellite into its orbit around the earth



h → height of the orbit

M → mass of the earth

R → Radius of earth

Distance between point mass and centre of satellite

$$F = \frac{mv_0^2}{R+h}$$

$$\frac{GM\cancel{m}}{(R+h)^{\cancel{2}}} = \frac{\cancel{m}v_0^2}{\cancel{(R+h)}}$$

$$\frac{GM}{(R+h)} = v_0^2$$

$$v_0^2 = \frac{gR^2}{(R+h)}$$

When satellite near by earth

$$v_0 = \sqrt{\frac{gR^2}{R+h}}$$

$$v_0 = \sqrt{\frac{gR^2}{R+0}}$$

$$v_0 = \sqrt{\frac{gR^2}{R}}$$

$$R = \sqrt{\frac{g}{(R+h)}}$$

$$v_0 = \sqrt{gR}$$

$$g = 9.8$$

$$R = 6400 \times 10^3 m$$

$$v_0 = \sqrt{9.8 \times 6400 \times 10^3}$$

$$= \sqrt{6272 \times 10^4}$$

$$= 79.19 \times 10^2 m/s$$

$$7.9 \times 10^3 m/s$$

$$v_0 = 7.9 \; KM/s.$$

Note: It is independent of mass of satellite

(2) It is depends on the mass and radius of the planet about which the satellite revolve

(3) Re

Relation between orbital velocity and escape speed

The escape speed of the earth is $V_e = \sqrt{2gR}$

The orbital velocity of the satellite around earth is $V_o = \sqrt{gR}$

$$V_e = \sqrt{2gR}$$

$$V_o = \sqrt{gR}$$

$$\frac{eq(1)}{eq(2)} = \frac{\sqrt{2gR}}{\sqrt{gR}} = \frac{V_e}{V_o}$$

$$\sqrt{2}\,\frac{\sqrt{gR}}{\sqrt{gR}} = \frac{V_e}{V_o}$$

$$\frac{\sqrt{2}}{1} = \frac{V_e}{V_o}$$

$\sqrt{2} : 1$ Ratio of V_e & V_o

$$V_e = \sqrt{2}V_o$$

$$V_e = 1.414 V_o$$

$$V_e = 11.2 \text{ km/s}$$

$$V_o = 7.9 \text{ km/s}$$

$$V_e > V_o$$

The escape velocity of the body from the earth surface is $\sqrt{2}$ times its velocity in a circular orbit just above the earth surface

Time period of satellite

It is the time taken by a satellite to complete one revolution around the earth

$$t = \frac{d}{v}$$

$$T = \frac{\text{Circumference of a circle}}{\text{orbital velocity}}$$

$$T = \frac{2\pi(R+h)}{\sqrt{\frac{GM}{(R+h)}}}$$

$$T = \frac{2\pi(R+h)}{\sqrt{\frac{gR^2}{(R+h)}}}$$

$h \to 0$

$$T = \frac{2\pi R}{\sqrt{gR}}$$

$$2\pi(R+h)\sqrt{\frac{R+h}{GM}}$$

$$= \frac{2\pi(R+h)(R+h)}{\sqrt{GM}}$$

$$T = \frac{2\pi(R+h)^{3/2}}{\sqrt{GM}}$$

$GM = gR^2$

$$T = \frac{2\pi(R+h)^{3/2}}{\sqrt{gR^2}}$$

$$T = 2\pi((R+h)^3$$

$$T = \frac{2\pi\sqrt{(R+h)^3}}{\sqrt{gR^2}}$$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

$h = 0$

$$T = 2\pi\sqrt{\frac{R^3}{gR^2}}$$

$$T = 2\pi\sqrt{\frac{R}{g}}$$

$R = 640 \times 10^4 m$

$g = 9.8$

$$T = 2 \times 3.14\sqrt{\frac{640 \times 10^4}{9.8}}$$

$$2 \times 3.14\sqrt{65.30 \times 10^4}$$

$= 2 \times 3.14 \times 8.08 \times 10^2$

$= 50.76 \times 10^2$ seconds

$= \frac{50.76 \times 10^2}{60}$

$= 0.846 \times 10^2$ minutes

$= 84.6$ minutes.

Energy of an orbitting satellite.

Consider a total energy of an orbitting satellite

Consider a satellite of mass m moving around the earth with velocity V_0 in an orbit of radius R. Because of gravitational pull of the earth, the satellite has potential energy which is given by $u = -\frac{GMm}{R}$

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2} m$

$$V_0 = \sqrt{\frac{GM}{r}}$$

$$K.E = \frac{1}{2} m \left(\sqrt{\frac{GM}{r}}\right)^2$$

$$K.E = \frac{1}{2} m \frac{GM}{r}$$

$$= \frac{GMm}{2r}$$

Total energy = KE + PE

$$\frac{GMm}{2r} + \left(-\frac{GMm}{r}\right]$$

$$\frac{GMm}{r} \left[\frac{1}{2} - 1\right]$$

$$TE = \frac{GMm}{r} \left[-\frac{1}{2}\right]$$

$$TE = -\frac{GMm}{2r}$$

$$TE = -\frac{GMm}{2(R+h)}$$

R → Radius

H → Height

If when h = 0 & near to the surface earth?

$$TE = \frac{GMm}{2(R+0)}$$

$$TE = -\frac{GMm}{2R}$$

16.09.25

Chapter

Mechanical Properties of Solids

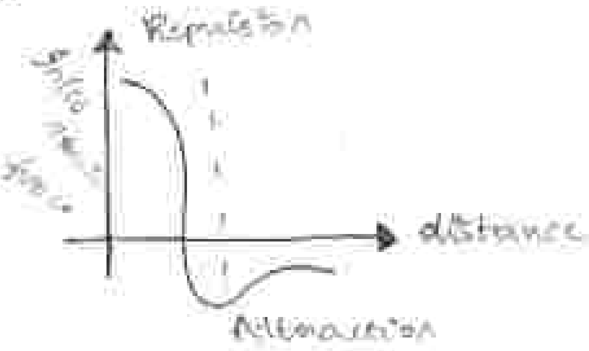
Elasticity deals with properties of material, its strength and ability to withstand against external forces which are acting on it.

Inter atomic force

. The force acting between the atoms due to electrostatic induction between the charges of the atom are called Interatomic force

. These are electric in nature

. The active Ef the distance between the two atoms is of the order

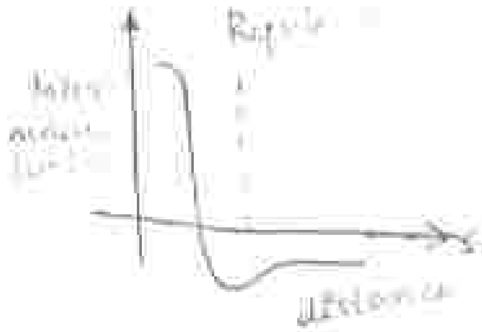


Inter molecular force

The force between the molecule due to electrostatic interaction between the charges of the molecule are called - Intermolecular force

These forces are active if the separation between two molecules is of the order of molecular size 10^{-9} m

The variation of Intermolecular force with distance



The force of attraction between the molecule where is inversely on the 4th power of intermolecular distance r.

$$F_a \propto \frac{1}{r^7}$$

$$F_a = \frac{-a}{r^7}$$

The negative sign indicates that the force is attractive in nature

when the distance between the molecules become less than R_0 the force between becomes repulsive in nature

Some Important definitions

Find the potential energy of a system of 4 particle placed at the vertices of a square of side l also obtain the potential at the centre of the square

$$PE_{ABCD} = -\frac{4GM^2}{l}$$

$$PE_{AC} = -\frac{2GM^2}{\sqrt{2}l}$$

$AC^2 = l^2 + l^2$

$AC^2 = \sqrt{2l^2}$

$= \sqrt{2}l$

$$PE_{total} = \frac{-4GM^2}{l} + \left(\frac{-2GM^2}{\sqrt{2}l}\right)$$

$$= -\frac{4GM^2}{l} - \frac{2GM^2}{\sqrt{2}l}$$

$$= \frac{GM^2}{l}\left[4 + \frac{2}{\sqrt{2}}\right]$$

$$= \frac{GM^2}{l}\left[\frac{4\sqrt{2} + 2}{\sqrt{2}}\right]$$

$$PE = \frac{2GM^2}{l}\left[2 + \frac{1}{\sqrt{2}}\right]$$

$$-\frac{2GM^2}{l}\left[2 + \frac{1}{1.41}\right]$$

$$= -\frac{2GM^2}{l^2}\left[2 + 0.70\right]$$

$$G = \frac{5.4Gm^2}{l^2}$$

$$G(3) = \frac{-GM^2}{\frac{\sqrt{2}\,l}{2}}$$

$$G(3) = \frac{-2GM^2}{\sqrt{2}l} \times 4$$

$$G(3) = \frac{-\sqrt{2}\cdot\sqrt{2}GM \times 4}{\sqrt{2}l}$$

$$G(3) = \frac{4\sqrt{2}\,2GM}{l}$$

2. If g on the surface of the earth is 9.8m Find its value at the depth of 3200km (radius of the earth = 6400

$$gd = g\left[1 - \frac{d}{R}\right] \quad \frac{3200\times10^3}{6400\times10^3}$$

$d = 3200$ km $= 3200$ m

$R = 6400$ km $= 6400$ m

$g = 9.8 m/s^2$

$$gd = 9.8\left[1 - \frac{3200}{6400}\right] = 9.8\left[1 - \frac{1}{2}\right]$$

$$9.8[0.5]$$

$$gd = 4.9 ms^{-2}$$

Find the value of g at a height Km from the surface of the earth [radius of the earth 6400]

$$g_h = g\left[1 - \frac{2h}{R}\right]$$

$$= 9.8\left[1 - \frac{2 \times 100 \times 10^3}{6400 \times 10^3}\right]$$

$$= 9.8\left[1 - \frac{1}{32}\right]$$

$$= 9.8\left[\frac{31}{32}\right]$$

$$= 9.8\left[0.96\right]$$

$$= 9.1 ms^{-2}$$

A satellite orbits the earth at a height of 400km above the surface. How much energy must be expanded to rocket the satellite out of the gravitational influence of earth? mass of the satellite is 200kg, mass of the earth 6×10^{24} kg, radius of the earth $= 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}\ Nm^2 kg^{-2}$

$$T = -\frac{GMm}{(R+h)} + \frac{1}{2}mv^2$$

$$= -\frac{GMm}{(R+h)} + \frac{1}{2}m \cdot \frac{GM}{R+h}$$

$$= -\frac{GMm}{(R+h)} + \frac{GMm}{2(R+h)}$$

$\frac{GMm}{R+h}\left(-1+\frac{1}{2}\right)$

$= \frac{-1\,GMm}{2R+h}$

Total energy = - (Total energy of the satellite)

$= \frac{-(-6.67\times10^{-11}\times6\times10^{24}\times200)}{2(6.4\times10^{6}+400\times10^{3})}$

$= \frac{6.67\times10^{-11}\times\cancel{1200}^{600}\times10^{24}}{\cancel{2}(6.8\times10^{6})}$

$= \frac{6.67\times10^{-17}\times600\times10^{24}}{6.8}$

$= 0.98\times600\times10^{7}$

$E = 588\times10^{7}\,J$

$= 5.8\times10^{9}\,J$

A geostationary satellite orbits the earth at a height of nearly 36000 km. from the surface of earth. What is the potential due to earth's gravity at the site of this satellite? [Take the potential energy at ∞ to be 0.] mass of the earth 6×10^{24} Kg, radius = 6400 Km, $G = 6.67\times10^{-11}\,Nm^2Kg^{-2}$

$V = -\frac{GM}{(R+h)}$

$V = -\frac{6.67\times10^{-11}\times6\times10^{24}}{(6400+36000)}$

$= \dfrac{6.67 \times 6 \times 10^{24-11}}{42400}$

$= \dfrac{40.02 \times 10^{13}}{42400}$

$= 0.00094 \times 10^{13}$

$= 9.4 \times 10^{6}\ J/Kg$

continuation

Some Important definition

(i) Deforming force :- when an external force is applied on an body & not which is not free to move, the molecules of the body are forced to undergo a change in their relative position.

(or)

The body get change in length or volume or shape by the external force such a body is said to be Deform.

(ii) Restoring force :- The force developed within the body on account of relative molecular displacement is called internal force or elastic force.

(iii) Rigid body :- The body is said to be rigid if the relative positions of its particles remains unchanged inspite of any amount of deforming force.

(iv) Elasticity :- It is the property of material of a body by virtue of which the body regains its original length volume and shape after the deforming forces have been removed

Plasticity: The property of a material of a body the virtue of which it does not regain its original shape and size even after the removal of deforming force is called plasticity.

Elastic behaviour of solid

In a solid each atom or molecule is surrounded by neighbouring atoms or molecules. Those are bounded together by inter atomic or inter molecular forces

(2) Stress :- The internal restoring force acting per unit area of cross section of the deformed body

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area of cross section}}$$

S.I unit : Nm^{-2}

Dimensional formula : $[ML^{-1}T^{-2}]$

Stress is a tensor quantity

Types of stress

1. longitudinal stress.

2. Bulk stress

3. Shearing or tangential stress

Properties of fluid

Fluid :
Examples of Fluid
(1) Oil
(2) Water

It has less intermolecular

It has less Interatomic and it is partially packed.

Pressure = $\frac{F}{\downarrow A \uparrow}$ → Pascal [SI unit]

$P_{ext} = P_A + P_B + P_C$

To calculate Average :

$$P_{avg} = \frac{\frac{P_1 + P_2 + P_3}{3}}{A}$$

Problem : The two thigh bones each of cross sectional area 10cm² support the upper part of human body of mass 40kg estimate the average pressure sustained by the femurs

$P_{av} = \frac{F}{A}$

$A = 10cm^2$

$2 \times 10 (10^{-2})^2$

20×10^{-4}.

F = 40N

$F = mg = 40 \times 10 = 400$

$\frac{F}{A} = \frac{400}{20 \times 10^{-4}}$

$= 2 \times 10^5 Nm^{-2}$

Pascal's law

Variation of pressure in depth

$w = mg$

$w = Ah\rho g$

$\rho = \frac{m}{V}$

$m = \rho V$

$m = Ah\rho$

Upward direction = downward direction

$F_2 = F_1 + mg$

$F_2 - F_1 = mg$

$P_2A - P_1A = mg$

$(P_2 - P_1)A = Ah\rho g$

$P_2 - P_1 = h\rho g$

$P - P_2 = h\rho g$

$P = P_a + h\rho g$

P_2 = Pressure

P_1 – Atmospheric pressure

$P_1 = P_a$

$P_1 = \frac{F}{A}$

$F_1 = P_1A$

$F_2 = P_2A$

Hydrostatic Paradox

A pressure in a horizontal fluid is proportional to the vertical distance of the surface of the fluid

1 atm = 1.03×10^5 Pascal

Gauge pressure

Gauge pressure is relative to atmospheric pressure above the atmosphere is positive below the atmosphere it is negative

Note : 1 torr → 133 pa

1 bar → 10^5 Pa

Atmospheric pressure :

(a) Barometer

It is a device used to measure atmospheric pressure the pressure at point A is zero because of vaccum the pressure at point B and point A are equal suppose point B is at depth H below A and ρ be the density of a mercury

$P_B = P_C = P_a \rightarrow$ Atmospheric pressure

$P = P_a + \rho gh$

$P = P_a + \rho gh$

$P_B = P_A + \rho gH$

$P_a = 0 + \rho gH$

$P_a = \rho gH$

(b) Manometer

It is a device used to measure a pressure of any gas contained in a closed vessel.

It consist of a U-tube having some liquid. One end is the open tube the atm and the other end is connected to the vessel connected to the gas

The pressure of enclosed glass = $P_A = P_B$.

$P_A = P_B$

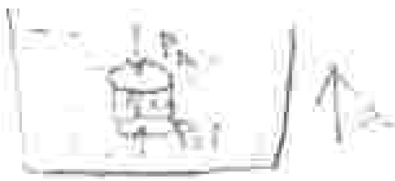
$P_c = P_a$

$P_A = P_c + \rho g h$

$P_B = P_a + \rho g h$

$P = P_a + \rho g h$.

A Pressure difference when liquid is accelerating in vertical direction



From $F = ma$

$F = F_2 - F_1$

$ma = P_2A - (mg + P_1A)$

$hA\rho a = P_2A - hA\rho g - P_1A$ $m = V\rho$ $= hA\rho$

$hA\rho a + hA\rho g = P_2A - P_1A$

$(P_2 - P_1)\cancel{A} = h\cancel{A}\rho(a+g)$

$(P_2 - P_1) = h\rho(a+g)$

Downward direction

$(P_2 - P_1) = h\rho(g-a)$

(i) what is the pressure of a swimmer 10m below the surface of a lang lake

Density = $1000 kgm^{-3}$

Atmospheric pressure = 1.01×10^5

$P_A + \rho g h$

$= 1.01 \times 10^5 Pa + 1000 Kgm^{-3} \times 10 ms^{-2} \times 10m$

$= 2.01 \times 10^5 Pa$

or 2atm

The Density of a atmosphere at sea level is $1.29 Kg/M^3$. Assume that it does not change with Altitude. Then how high would the atmosphere extend?

$\rho g h$

$1.29 kg m^{-3} \times 9.8 ms^{-2} = 1.01 \times 10^5$

$h = \frac{1.01 \times 10^5}{1.29 \times 9.8}$

At a depth of 1000m in an ocean

(a) what is the absolute pressure

(b) what is the gauge pressure

(c) Find the force acting on the window of area 20cm x 20cm of a submarine at this depth. the interior which is maintained at sea level atmospheric pressure (The density of sea water is $1.03 \times 10^3 Kgm^{-3}$ $g = 10ms^{-2}$

(a) $P_a + \rho g h$

$= 1.01 \times 10^5 Pa + 1.03 \times 10^3 \times 10 \times 1000 m$

$= 1.01 \times 10^5 + 1.03 \times 10^3 \times 10000$

$= 104.01 \times 10^5$

$= 104$ atm

(b) $P_g = P - P_a = \rho g h$

$= 1.03 \times 10^3 \times 10 \times 1000$

$= 1.03 \times 10000 \times 10^3$

$= 1.030000 \times 10^5$

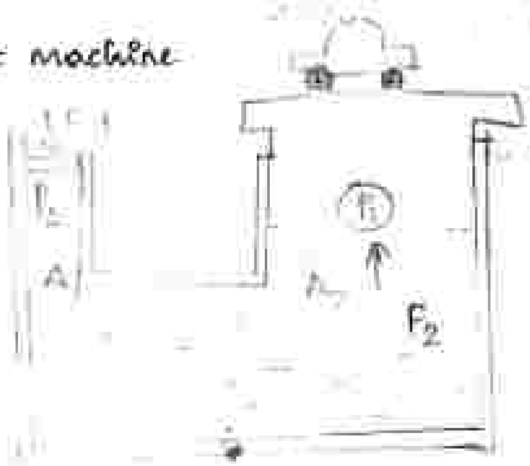
$= 1.03$ atm

(c) $P = P_a + \rho g h$

$= 103 \times 10^5 Pa \times 0.04$

$= 4.12 \times 10^5 N$

Hydraulic machine



Used in

* Hydraulic lift
* Hydraulic Brake

$P_1 = \frac{F_1}{A_1}$ $\quad P_2 = \frac{F_2}{A_2}$

$P_1 = P_2$

$\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$F_1 = \frac{F_2}{A_2} \cdot A_1$

$P_1 \cancel{A_1} = \frac{F_2 \cdot \cancel{A_1}}{A_2}$

$P_1 = \frac{F_2}{A_2}$

$\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$A_2 \cdot \frac{F_1}{A_1} = F_2$

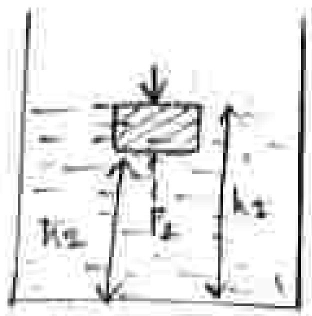
$\cancel{A_2} \frac{F_1}{A_1} = P\cancel{A_2}$

$P = \frac{F_1}{A_1}$

If an external pressure is applied to an enclosed fluid it is transmitted to every point in the fluid, and undiminished this is known as Pascal's law

Force of Buoyancy

$P = \frac{F}{A}$



$F_1 = P_1 A \quad F_2 = P_2 A$

$F_1 = h_1 g \rho A \quad F_2 = h_2 g \rho A$

$F_B = F_2 - F_1$

$= h_2 g \rho A - h_1 g \rho A$

$= g \rho A (h_2 - h_1)$

$F_B = g \rho V$

$F_B \rightarrow$ density of materials

$F_B = g \rho V \frac{\sigma}{\sigma}$ $\qquad \sigma = \frac{m}{V}$

$= \frac{g \rho m}{\sigma}$ $\qquad \sigma V = m$

$F_B = mg \frac{\rho}{\sigma}$

Stream line flow :-

The flow of a fluid is said to be steady, orderly. is called stream line flow [laminar flow] Every particle of the fluid flows exactly the path of its proceeding particle and has the same velocity

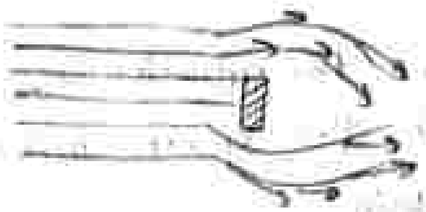
Properties :-

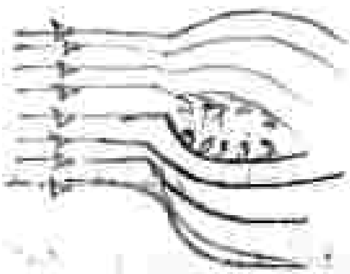
- The Tangent at any point of the stream line gives the direction of velocity of the liquid at that point.
- A stream line flow is not necessarily an ideal flow It is not a necessarily incompressible fluid

Turbulant flow:-

The flow of a fluid is said to be unsteady, turbulant

Every particle of a ffuid not following exactly the path of its proceeding particle





Equation of continuity

At point A = $a_1 v_1 s_1$

~~At point B = $a_2 v_2$~~

The rate of mass of flow of an ideal, fluid at every cross section remains constant

Here

$a_1, a_2 \rightarrow$ area of cross section

V_1 and $V_2 \rightarrow$ Velocity

At point A = $a_1 v_1 \rho_1 \rightarrow (1)$

At point B = $a_2 v_2 \rho_2$

$$eq(1) = eq(2)$$

$$a_1 v_1 \rho_1 = a_2 v_2 \rho_2$$

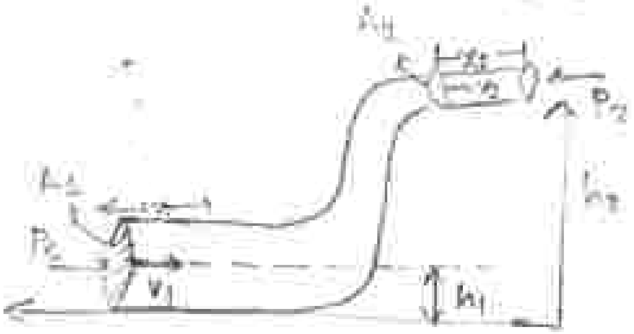
$$\frac{a_1}{a_2} = \frac{v_1}{v_2}$$

$$\boxed{av = \text{constant}}$$

This equation is called the equation of continuity for the stream line flow of an incompressible and non viscous fluid

Bernoulli's Theorm.

When a non-viscous incompressible fluid flows steadily. The sum of Pressure, kinetic energy and potential energy for unit volume of the fluid remains constant at all points in the path of the flow



P + KE + PE = constant

Consider the flow of fluid is initially lieing between B and D. let v_1, v_2 are the speed of the fluid at B and D. In a small interval of time ΔT, let the fluid at B move to C and fluid at D move to E since ΔT is very small interval of time, it can assume that the area of cross section of region BC is uniform and also in the region DE

Distance of the fluid from A to B

$x_1 = v_1 \Delta t$

Distance of the fluid from C to D

$x_2 = v_2 \Delta t$

workdone = F × displacement

$= P_1 A_1 \times \Delta x$

At A to B

$w_1 = P_1 A_1 v_1 \Delta t$

$w_1 = P_1 \Delta V$

at C to D

$W_2 = -P_2 A_2 v_2 \Delta t$

$W_2 = -P_2 \Delta V$ $\qquad$ $\Delta V \rightarrow$ volume per unit time

net workdone

$w = w_1 + w_2$

$w = P_1 \Delta V + (-P_2 \Delta V)$

$w = P_1 \Delta V - P_2 \Delta V$

$w = (P_1 - P_2) \Delta V \rightarrow (1)$

$KE = \frac{1}{2} m v^2$

$PE = mgh$

Rate of mass of the fluid

$\Delta m = \rho A \Delta t \cdot v$

$\Delta m = \rho \Delta V \rightarrow (3)$

Now, $KE = \frac{1}{2} \Delta m (V_2 - V_1)^2$

$KE = \frac{1}{2} \rho \Delta V (V_2 - V_1)^2 \rightarrow (3)$

Now, $PE = \Delta m g h$

$= \Delta m g (h_2 - h_1)$

$= \rho \Delta V g (h_2 - h_1) \rightarrow (4)$

From work energy theorm

$W = KE + PE$

$(P_2 - P_1) \Delta V = \frac{1}{2} \rho \Delta V (V_2 - V_1)^2 + \rho \underline{\Delta V} g (h_2 - h_1)$

$(P_1 - P_2) \Delta V = \underline{\Delta V} (\frac{1}{2} \rho (V_2 - V_1)^2 + \rho g (h_2 - h_1))$

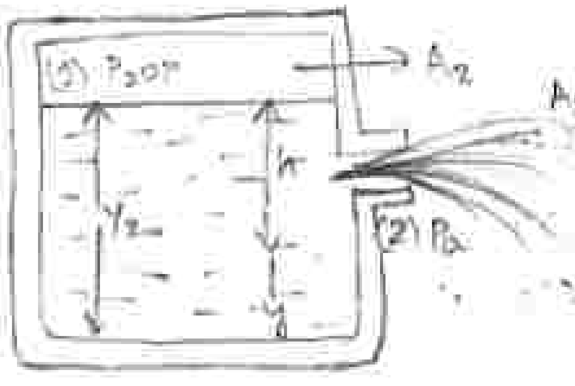
$P_1 - P_2 = \frac{1}{2} \rho (V_2 - V_1)^2 + \rho g (h_2 - h_1)$

$P_1 - P_2 = \frac{1}{2} \rho V_2^2 - \frac{1}{2} \rho V_1^2 + \rho g h_2 - \rho g h_1$

$P_1 + \frac{1}{2} \rho V_1^2 + \rho g h = P_2 + \frac{1}{2} \rho V_2^2 + \rho g h_2$

$P + \frac{1}{2} \rho V^2 + \rho g h = \text{constant}$

Speed of efflux - Torricell's.



Equation of continuity

$A_1 V_1 = A_2 V_2$

$\frac{A_1}{A_2} = \frac{V_2}{V_1}$

$V_1 \frac{A_1}{A_2} = V_2$

$$\left[\frac{V_1 A_1}{A_2} = 0\right] \quad [V_2 = 0]$$

Applying Bernoulli's theorem at 2 points

$$P_a + \frac{1}{2}\rho V_1^2 + \rho g y_1 = P + \rho g y_2$$

$y_2 - y_1 = h$

$$P_a + \frac{1}{2}\rho V^2 = P + \rho g y_2 - \rho g y$$

$$P_a + \frac{1}{2}\rho V_1^2 = P + \rho g (y_2 - y_1)$$

$$P_a + \frac{1}{2}\rho V_1^2 = P + \rho g h$$

$$P - P + \frac{1}{2}\rho V_1^2 = \rho g h$$

$$\rho V_1^2 = 2\rho g h$$

$$V_1^2 = \frac{2\rho g h}{\rho}$$

$$V_1 = \sqrt{2gh}$$

Applications of Bernoli's theorm

(i) Attraction between two closely parallel moving boats.

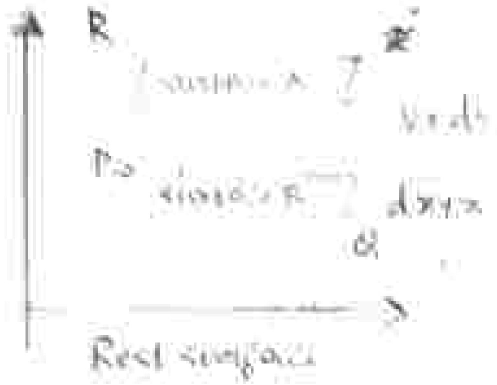
(ii) working of an aeroplane wing

(iii) Action of sprayer or atomizer

(iv) spinning ball

Viscosity :-

The property of the liquid by the virtue of which it oppose the Relative motion between its adjacent layers is known as Viscosity



Consider two layers PQ and RS of the fluid at the distance x and $x+dx$ from the fixed surface.

$F \propto A$ (Area of cross section) $\rightarrow (1)$

$F \propto$ Velocity gradient $\rightarrow (2)$

From eq(1) and (2)

$F \propto A \times$ Velocity gradient

$F = -\eta A \times$ velocity gradient

$$\boxed{F = -\eta A \times \frac{dv}{dx}}$$

Co-efficient of dynamic viscosity

Tangential force per unit area of the layer, required to maintain velocity gradient

Depends : Nature of the fluid

(ii) Independent of area considered

(iii) Velocity gradient

SI unit = Poisseuille [PI]

1 PI = 10 Poise

CGS unit : dyne/cm^2

MKS $\rightarrow$ PI $\rightarrow$ Ns/m^2

dimensional formula : $\frac{kgms^{-1}}{m^2} = \frac{kgs^{-1}}{m} = [ML^{-1}T^{-1}]$

$$F = -\eta A \frac{dv}{dx}$$

$$MLT^{-2} = ML^{-1}T^{-1} \times L^2 \times \frac{LT^{-1}}{L}$$

$$= ML^{-1}T^{-2}L^2$$

$$= M^1L^{+1}T^{-2}$$

Stoke's law

It shows that a sphere of radius (r) are moving with relative velocity V through a stationary medium of viscosity η & then the viscos force acting on the sphere is

$$F = 6\pi\eta r V$$

$$F = MLT^{-2}$$

$$MLT^{-2} = K\eta^a r^b v^c$$

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^a [L]^b [LT^{-1}]^c$$

$$= [M^a L^{-a} T^{-a}][L^b][L^c T^{-c}]$$

$$[MLT^{-2}] = M^a L^{-a+b+c} T^{-a-c}$$

$M = a = 1$

$L = -a+b+c = 1$

$-1+b+c = 1$

$b+c = 2 \rightarrow (1)$

$$F = K\eta^1 r^1 v^1$$

$-2 = -a-c$

$-2 = -1-c$

$-2+1 = -c$

$-1 = -c$

$c = 1$

Surface tension

A liquid surface just like a stretched membrane is capable of carrying a small loads and tends to acquire minimum surface area this property of liquid is called surface tension

Cohesive and adhesive force

Cohesive force: Force of attraction between molecules of small substances

Eg: water to fingers
Paint to the wall

Adhesive force: The force of attraction between molecules of different substances

Eg: Force between molecules gum and paper

Surface tension $S = \frac{\text{Force}}{\text{length}}$

SI unit of surface tension $= Nm^{-1}$

Applications:-

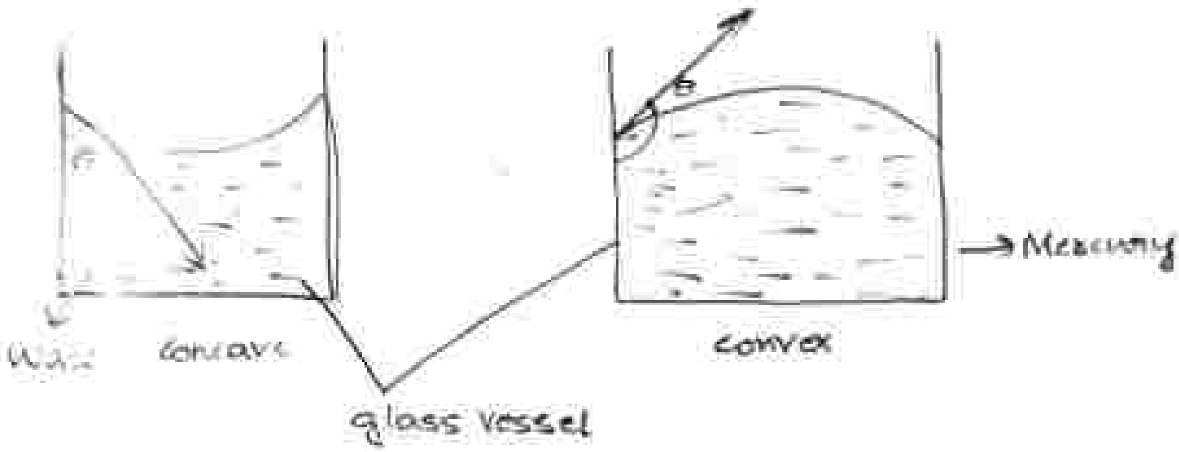
- Manufacture of lead shots
- Oil and paints

Angle of contact: The angle 'θ' between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid.

Depends:

- Nature of the solid and the fluid
- Temperature of the liquid

Formation of Angle of liquid in different material



Silver Vessel